

*Evaluation of the Implementation of an Abstract Interpretation Algorithm using Tabled CLP **

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Abstract

CiaoPP is an analyzer and optimizer for logic programs, part of the Ciao Prolog system. It includes PLAI, a fixpoint algorithm for the abstract interpretation of logic programs which we adapt to use *tabled constraint logic programming*. In this adaptation, the tabling engine drives the fixpoint computation, while the constraint solver handles the LUB of the abstract substitutions of different clauses. That simplifies the code and improves performance, since termination, dependencies, and some crucial operations (e.g., branch switching and resumption) are directly handled by the tabling engine. Determining whether the fixpoint has been reached uses *semantic equivalence*, which can decide that two syntactically different abstract substitutions represent the same element in the abstract domain. Therefore, the tabling analyzer can reuse answers in more cases than an analyzer using syntactical equality. This helps achieve better performance, even taking into account the additional cost associated to these checks. Our implementation is based on the TCLP framework available in Ciao Prolog and is one-third the size of the initial fixpoint implementation in CiaoPP. Its performance has been evaluated by analyzing several programs using different abstract domains.

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1 Introduction

Tabling (Tamaki and Sato 1986; Warren 1992) is an execution strategy for logic programs that suspends repeated calls which could cause infinite loops. Answers from non-looping branches are used to resume suspended calls which can, in turn, generate more answers and resume other suspended calls. Only new answers are saved, and evaluation finishes when no new answers can be generated. Tabled evaluation always terminates for calls/programs with the bounded term depth property (i.e., they can only generate terms with a fixed finite depth) and can improve efficiency for terminating programs which repeat computations, as it automatically implements a variant of dynamic programming. Tabling has been successfully applied in a variety of contexts, including deductive databases, program analysis, semantic Web reasoning, and model checking.

Constraint Logic Programming (CLP) (Jaffar and Maher 1994) extends Logic Programming (LP) with variables that can belong to arbitrary constraint domains and the ability to incrementally solve equations involving these variables. CLP brings additional expressive power

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to LP, since constraints can very concisely capture complex relationships. Also, shifting from “generate-and-test” to “constraint-and-generate” patterns reduces the search tree and therefore brings additional performance, even if constraint solving is in general more expensive than unification.

The integration of tabling and constraint solvers makes it possible to exploit their synergy in several application fields: abstract interpretation (Swift and Warren 2010), reasoning on ontologies, and constraint-based verification (Gange et al. 2013). In this paper we use Mod TCLP (Arias and Carro 2019a) to adapt PLAI, the fixpoint algorithm implemented in the program analysis, optimization, and transformation tool CiaoPP (Hermenegildo et al. 2012; Hermenegildo et al. 2005). The re-implementation of PLAI uses tabling to reach the fixpoint (following ideas similar to (Kanamori and Kawamura 1993; Janssens and Sagonas 1998)), incremental aggregation techniques (Guo and Gupta 2008; Zhou et al. 2010; Swift and Warren 2010; Arias and Carro 2019b) to join the answers, by discarding the more particular ones, and call entailment checks (Chico de Guzmán et al. 2012; Arias and Carro 2019a) to detect repeated calls (in order to suspend execution to reuse answers from previous calls), thereby speeding up convergence. The resulting code space is reduced to one third and, consequently, increases the maintainability of the abstract interpreter.

2 Related Work

Abstract interpretation has always been seen as one of the most clear applications of tabled logic programming. It requires a fixpoint procedure, often implemented using memo tables and dependency tracking, which play a role very similar to the internal data structures that tabling engines need to detect repeated calls, store and reuse answers, and check for termination.

The relationship between abstract interpretation and tabling was recognized very early. *Extension tables* (Dietrich 1987) were proposed to record results from the execution of predicates and turn intensional definitions into extensional definitions. Their applications included “improving the termination and completeness characteristics of depth-first evaluation strategies in the presence of recursion”. The idea of extension tables were applied as the embryo of SLG resolution and the XSB system. At the same time, abstract interpretation was then viewed as inefficient, and as part of the efforts to make it a practical technique to implement analyzers, tables, but also other ideas such as dependency tracking, were used (Warren et al. 1988), thus making it clear that a common underlying technology could be used in both types of systems.

The next step was to use these components, independently available in tabling systems, to explore how they could be used to build abstract interpreters. Earlier work (Kanamori and Kawamura 1993) explored the possibilities offered by OLDT (Tamaki and Sato 1986) to implement abstract interpretation. Using type inference as the guiding example, it suggests certain changes to OLDT and concludes that it is feasible to do abstract interpretation with OLDT. The paper neither describes an implementation nor reports performance, but it states that the abstract interpreter was implemented and was available. In (Warren 1999) an abstract interpreter written in XSB is presented as one of the applications of tabled Prolog.

However, surprisingly few examples of abstract interpreters implemented using tabling have been presented and evaluated w.r.t. implementations without tabling. One of them is a framework (Janssens and Sagonas 1998) based on abstract compilation that executes the abstract version of the program under analysis, together with domain-dependent abstract operations, which is evaluated using the tabling system XSB and compared with the AMAI and PLAI sys-

tems (Janssens et al. 1995; Muthukumar and Hermenegildo 1992). Both systems use abstract interpreters written in Prolog without tabling, but they rely on very different underlying technologies, and with different representations for the abstract domains. From that evaluation, the paper concludes that tabling is a viable infrastructure for abstract interpretation, but concedes that the PLAI fixpoint algorithm was the most efficient abstract interpreter for logic programming available at the moment. The very different underlying infrastructure makes it difficult to use these results to draw meaningful conclusions.

On the other hand, abstract interpretation has been used as a benchmark to compare different implementations and/or scheduling strategies of tabling (Demoen and Sagonas 1998; Freire et al. 2001). Advanced tabled systems and techniques have been proposed to implement more efficient abstract interpreters by using the *least upper bound* operator (Schrijvers et al. 2008) to combine answers, numeric constraint solvers (Chico de Guzmán et al. 2012) to implement the Octagon domain, and the *partial order answer subsumption with abstraction* (Swift and Warren 2010) for cases where, e.g., the program computed does not have a finite model. However, none of them reports performance evaluation against other frameworks.

In this paper we started with PLAI, the state-of-the-art abstract interpreter used by CiaoPP, and re-implemented its fixpoint procedure in Tabled CLP preserving the interface with the rest of the system. Therefore, we can compare some indicators of code complexity (e.g., comparing lines of code, with the assumption that the tabled version is essentially a subset of the original version) and performance on a completely equal footing. This is, to our best knowledge, the first comparison that has these characteristics.

3 Background

In this section we briefly describe Mod TCLP (Arias and Carro 2019a), a generic interface that facilitates the integration of constraint solvers with the tabling engine in Ciao, Aggregate-TCLP (Arias and Carro 2019b), a framework implemented on top of Mod TCLP to incrementally compute lattice-based aggregates, and PLAI, the fixpoint algorithm used by CiaoPP.

3.1 The Mod TCLP framework

Tabled Logic Programming with Constraints (TCLP) (Arias and Carro 2019a; Schrijvers et al. 2008; Cui and Warren 2000) improves program expressiveness and, in many cases, efficiency and termination properties. Let us consider a program to compute distances between nodes in a graph written using tabling (Fig. 1, left). The query $?- \text{dist}(a, Y, D), D < K.$ would loop under SLD due to the left-recursive rule, while it would terminate under tabling for acyclic graphs.

Tabling records the first occurrence of each call to a tabled predicate (the *generator*) and its answers. In variant tabling (the most usual form of tabling), when a call is found to be equal, modulo variable renaming, to a previous generator, the execution of the call is suspended and it is flagged as a *consumer* of the generator. For example $\text{dist}(a, Y, D)$ is a variant of $\text{dist}(a, Z, D)$ if Y and Z are free variables. Upon suspension, execution switches to evaluating another untried branch. A branch which does not suspend can generate answers for the initial goal. When a generator finitely finishes exploring all the clauses and all answers are collected, the consumers that depend on it are resumed and fed with the answers of the generator. This may make generators produce new answers which can in turn resume more consumers. This process finishes when

<pre> 1 :- table dist/3. 2 3 dist(X,Y,D) :- 4 dist(X,Z,D1), 5 edge(Z,Y,D2), 6 D is D1+D2. 7 dist(X,Y,D) :- 8 edge(X,Y,D). </pre>	<pre> 1 :- table dist/3. 2 3 dist(X,Y,D) :- 4 D1 #> 0, D2 #> 0, 5 D #= D1+D2, 6 dist(X,Z,D1), 7 edge(Z,Y,D2). 8 dist(X,Y,D) :- 9 edge(X,Y,D). </pre>	<pre> 1 :- table dist(_,_,min). 2 3 dist(X,Y,D) :- 4 dist(X,Z,D1), 5 edge(Z,Y,D2), 6 D is D1+D2. 7 dist(X,Y,D) :- 8 edge(X,Y,D). </pre>
(a) Tabling	(b) TCLP	(c) Aggregate-TCLP

Fig. 1: Distance traversal in a graph. Note: The symbols #> and #= are (in)equalities in CLP.

no new answers can be generated — i.e., a fixpoint has been reached. Tabling is sound and, for programs with a finite Herbrand model, complete (and, therefore, it always finishes in these cases).

However, in a cyclic graph, `dist/3` has an infinite Herbrand model: every cycle can be traversed repeatedly and create paths of increasing length. Therefore, the previous query `?- dist(a,Y,D), D < K` will not terminate under variant tabling, although the query as a whole has a finite model.

On the other hand, if the integration of tabling and CLP (Fig. 1, center) uses *constraint entailment* (Chico de Guzmán et al. 2012), calls to `dist/3` will suspend if there are previous similar calls that are more general, and only the most general answers will be kept. The query `?- D #< K, dist(a,Y,D)` terminates under TCLP because by placing the constraint `D #< K` before `dist(a,Y,D)`, the search is pruned when the values in `D` are larger than or equal to `K`.

This illustrates the main idea underlying the use of entailment (\sqsubseteq) in TCLP: more particular calls (consumers) can suspend and later reuse the answers collected by more general calls (generators). In order to make this entailment relationship explicit, we will represent a TCLP goal as $\langle g, c_g \rangle$ where g is the call (a literal) and c_g is the projection of the current constraint store onto the variables of the call. For example, $\langle \text{dist}(a, Y, D), D > 0 \wedge D < 75 \rangle$ entails the goal $\langle \text{dist}(a, Y, D), D < 150 \rangle$ because $(D > 0 \wedge D < 75) \sqsubseteq D < 150$. The latter is therefore more general (i.e., it is a generator) than the former (a consumer). All the solutions of a consumer are solutions for its generator, since the space of solutions of the consumer is a subset of that of the generator. However, not all answers from a generator are valid for its consumers. For example $Y = b \wedge D > 125 \wedge D < 135$ is a solution for our generator, but not for our consumer, since the consumer call was made under a constraint store more restrictive than the generator. Therefore, the tabling engine has to filter, via the constraint solver, the answers from the generator that are consistent w.r.t. the constraint store of the consumer.

Additionally, the Mod TCLP framework (Arias and Carro 2019a) has been used to implement in Ciao a framework, called Aggregate-TCLP (Arias and Carro 2019b), that incrementally computes aggregates for elements in a lattice. The Aggregate-TCLP framework uses the entailment and join relations in a lattice to define and compute aggregates, and to decide whether some atom is compatible with (i.e., entails) the aggregate. For example, the directive `:- table dist(_,_,min)` (Fig. 1, right), specifies the (aggregate) mode `min` for the third argument. The query `?- dist(a,Y,D)` will in this case terminate because only the shortest distance between two nodes found at every moment is kept, and it will be returned in `D` as a result of the evaluation of the initial call. Other tabling engines implement *answer subsumption* (Swift and Warren

2010) or a restricted form of it via *mode-directed tabling* (Guo and Gupta 2008; Zhou et al. 2010; Wielemaker et al. 2012; Santos Costa et al. 2012), that can be used to compute aggregates. However, answer subsumption, as implemented in XSB, assumes answers to be safe (i.e., ground) and works on non-ground answers only in some cases, so it would in principle not be applicable when answers are constraints. Answer subsumption also performs subsumption only on answers, while Aggregate-TCLP can in addition check entailment for calls. In the case of the TCLP implementation of the abstract interpreter, this makes it possible to reuse answers obtained from calls semantically equivalent (i.e., calls whose associated abstract substitutions differ, but that still represent the same object in the lattice) and/or more general (i.e., that represent an element higher in the lattice hierarchy). Note that in our benchmarks we are using semantic equivalence, since using entailment to detect more general calls would cause a loss of precision as the domains we are using are non-relational. Last, answer subsumption does not provide the freedom to be used with aggregates that cannot be expressed in terms of a lattice, such as $\text{sum}/3$, which (Arias and Carro 2019b) can work around.

3.2 The PLAI algorithm

We assume that the reader is familiar with the basic principles of abstract interpretation (Cousot and Cousot 1977; Bruynooghe 1991; Nielson et al. 2005). The PLAI algorithm used by the abstract interpreter of CiaoPP for static analysis extends the fixpoint algorithms proposed by (Bruynooghe 1991) with the optimizations described in (Muthukumar and Hermenegildo 1990). In logic programming, all possible concrete substitutions in the program (i.e., terms to which the variables in that program will be bound at run-time for a given query) can be infinite, which gives rise to an infinite execution tree. The core idea of PLAI is to represent this infinite execution tree by an abstract and-or tree using abstract substitutions to finitely represent the possibly infinite sets of substitutions in the concrete domain. The set of all possible abstract substitutions that a variable can be bound to is the *abstract domain* which is usually a complete lattice (or a complete partial order of finite height).

Domains in PLAI PLAI is domain-independent: new abstract domains can be easily implemented and integrated by using a common interface. The operations required by the domain interface are:

- $\lambda' \sqcup \lambda''$, which gives the LUB of the abstract substitutions λ' and λ'' . The LUB operation is defined in terms of the \sqsubseteq relation of the abstract domain.
- $\text{call_to_entry}(p(\vec{u}), C, \lambda)$, where C is a clause and $p(\vec{u})$ is a call. It gives an abstract substitution describing the effects on $\text{vars}(C)$ of unifying $p(\vec{u})$ with $\text{head}(C)$ given an abstract substitution λ for the variables in \vec{u} .
- $\text{exit_to_success}(\lambda, p(\vec{u}), C, \beta)$ which returns an abstract substitution describing the effect of execution $p(\vec{u})$ against clause C . For this, the variables of the abstract substitution β are renamed taking into account the unification with the terms in $\text{head}(C)$ and the variables in $p(\vec{u})$, and a new abstract substitution is returned updating λ with the new information.
- $\text{extend}(\lambda, \lambda')$ which extends abstract substitution λ to incorporate the information in λ' in a way that it is still consistent.
- $\text{project_in}(\vec{u}, \lambda)$ which extends the abstract substitution λ so that it refers to all the variables in \vec{u} .

Algorithm 1: `entry_to_exit`: Compute exit substitution from entry substitution.

Data: A clause C of the form $h(\vec{u}) :- p_1(\vec{u}_1), \dots, p_m(\vec{u}_m)$; an entry substitution β_{entry}
Result: An exit substitution β_{exit}
 $\lambda_1 := \text{project_in}(\text{vars}(C), \beta_{entry});$
for $i := 1$ to m **do**
 $\lambda_{i+1} := \text{call_to_success}(p_i(\vec{u}_i), \lambda_i);$
return $\text{project_out}(\vec{u}, \lambda_{m+1});$

Algorithm 2: `call_to_success`: Compute success substitution from call substitution.

Data: A goal $p(\vec{u})$; an abstract call substitution λ_{call}
Result: A success substitution $\lambda_{success}$
 $\lambda_{proj} := \text{project_out}(\vec{u}, \lambda_{call});$
 $\lambda' := \perp;$
for each clause C which unifies with $p(\vec{u})$ **do**
 $\beta_{exit} := \text{entry_to_exit}(C, \text{call_to_entry}(p(\vec{u}), C, \lambda_{proj}));$
 $\lambda' := \lambda' \sqcup \text{exit_to_success}(\lambda_{proj}, p(\vec{u}), C, \beta_{exit});$
return $\text{extend}(\lambda_{call}, \lambda');$

- $\text{project_out}(\vec{u}, \lambda)$ which restricts the abstract substitution λ to refer only to the variables in \vec{u} .

For additional examples of abstract domains integrated in CiaoPP, we refer the reader to (Bueno et al. 2004; Muthukumar and Hermenegildo 1989; Vaucheret and Bueno 2002).

And-Or trees and substitutions In PLAI, the abstract and-or tree is constructed using a top-down driven strategy (instead of a bottom-up computation) so that the computation is restricted to what is required for the given query. In the resulting and-or tree, an *and-node* is a clause head h whose children are the literals in its body, p_1, \dots, p_n , and an *or-node* is a literal, p_i , whose children are the heads h_1, \dots, h_m of the clauses that unify with p_i . Its construction starts with the abstract call substitution for the query. Then, abstract substitutions at all points of the abstract and-or tree are computed and finally, the success substitution for the query is computed.

Inside a clause, abstract substitutions at every point are denoted depending on their position among its literals. Given a clause $h :- p_1, \dots, p_n$, let λ_i and λ_{i+1} be the abstract substitutions to the left and right of the subgoal p_i , $1 \leq i \leq n$. Then, λ_i and λ_{i+1} are, respectively, the *abstract call substitution* and the *abstract success substitution* for the subgoal p_i . The projection of λ_1 on $\text{vars}(h)$ is the *abstract entry substitution*, β_{entry} , of the given clause, and, similarly, the projection of λ_{n+1} on $\text{vars}(h)$ is its *abstract exit substitution*, β_{exit} . The abstract substitutions for a clause are computed as follows:

- Exit substitution from the entry substitution (Algorithm 1): Given a clause $h :- p_1, \dots, p_n$ and an entry substitution β_{entry} for the clause head h , the call substitution λ_1 for p_1 is computed by simply adding to β_{entry} an abstraction for the variables in the clause that do not appear in the head. The success substitution for p_1 is λ_2 , and it is computed as explained below (essentially, by repeating this same process for the clauses which unify

with p_1). $\lambda_3, \dots, \lambda_{n+1}$ are computed similarly. The exit substitution β_{exit} for this clause is the projection of λ_{n+1} onto \vec{u} , the variables in h .

- Success substitution from the call substitution (Algorithm 2): Given a call substitution λ_{call} for a subgoal p , let h_1, \dots, h_m be the heads of clauses that unify with p . Compute the entry substitutions $\beta_{1_{entry}}, \dots, \beta_{m_{entry}}$ for these clauses. Compute their exit substitutions $\beta_{1_{exit}}, \dots, \beta_{m_{exit}}$ as explained above. Compute the success substitutions $\lambda_{1_{success}}, \dots, \lambda_{m_{success}}$ from the exit substitutions corresponding to these clauses. At this point, all different success substitutions can be considered for the rest of the analysis, or a single success substitution $\lambda_{success}$ for subgoal p computed by means of an aggregation operation for $\lambda_{1_{success}}, \dots, \lambda_{m_{success}}$. This aggregate is the least upper bound (LUB), denoted by \sqcup , of the abstract domain.

Note that these two procedures are mutually recursive and would not finish in case of mutually recursive calls. They merely describe how abstract substitutions are generated for the case of literals in a body (by carrying success abstract substitutions to call abstract substitutions) and how entry and exit substitutions of several clauses are composed together. For the general case of recursive predicates, where repeated calls and termination have to be detected, PLAI implements a fixpoint algorithm that we sketch below.

PLAI's fix point algorithm The core idea of PLAI's fixpoint algorithm (Muthukumar and Hermenegildo 1990) is that the subtree corresponding to the abstract interpretation of a node with a recursive predicate p should be finite. If the abstract domain is finite, a predicate p can only have a finite number of distinct call substitutions and therefore the subtree can only have a finite number of occurrences of nodes that have a variant of p and which themselves have subtrees. In addition to that, all other nodes in the subtree with the same predicate name p and with the same call substitutions (modulo variable renaming) use the approximate value of the success substitution computed previously for the root node of the subtree labeled with p , and hence they do not have any descendent nodes.

Based on this idea, the fixpoint algorithm iteratively refines the approximate values of the success substitution of the recursive predicate p as follows:

- First, it computes an approximate value of the projected success substitution using the LUB of the projected success substitutions corresponding to the **non-recursive** clauses of p . This provides an initial, hopefully non-empty, abstract substitution that is fast to compute (it does not need to check for repeated calls or termination) and accelerates the convergence of the fixpoint algorithm. In practice, it can be delegated to a specialized version of Algorithms 1 and 2 restricted to non-recursive calls / clauses. These can be determined beforehand by a reachability analysis based on strongly connected components.
- Then, it traverses the (finite) subtree corresponding to p in a depth-first fashion. When an entry-exit combination is needed for a call to p having the same call substitution (modulo variable renaming), the existing approximation is used. For a call to p with a different call substitution, a new (nested) fixpoint computation is started. When the analysis returns to the root of the subtree, the success substitution for p is updated as the LUB of the previous value and the value just computed from the recursive clauses of p .
- If there is a change in the success substitution for p , the depth-first traversal is restarted using the new success substitution, which is used for the subtree nodes corresponding to p that have a compatible call substitution. These depth-first traversal iterations can take place

only a bounded number of times, since the LUB operation is monotonic and the abstract substitutions form a lattice of finite height.¹ Therefore, a fixpoint will be reached in a finite number of steps.

- If there is no change in the success substitution for the root node of the subtree of p for a given call substitution, then the analysis of that subtree is complete (for that call substitution) and the fixpoint computation of the predicate p terminates.

For recursive predicates called from within recursive predicates, the dependencies between nested calls have to be recorded to restart the traversal of the subtrees containing predicate calls whose success substitution has been updated.

4 Implementations of the PLAI Algorithm: Prolog vs. Tabling

We will now describe more in depth how the PLAI algorithm is implemented in CiaoPP² and highlight the differences w.r.t. the version that uses Tabled CLP.

4.1 PLAI in CiaoPP

The implementation of `call_to_success` is the entry point, as it relates the entry and exit substitutions of a call (in particular, of the top-level call). During the analysis of a goal $p(\vec{u})$, and for each clause that unifies with $p(\vec{u})$, the predicate `call_to_success` invokes `entry_to_exit` which, for each subgoal in the body of the clause, invokes again `call_to_success`. The abstract interpreter is able to stop the evaluation of a part of the program and move to another part to evaluate calls to other predicates. The implementation of PLAI is optimized to accelerate the convergence of the fixpoint and reduce the computation by reusing previous results, among other techniques.

The PLAI algorithm is based on the construction of an and-or tree, described in Section 3.2, with the nodes representing the predicate calls visited during the analysis. To construct this tree, `call_to_success` identifies each goal with its corresponding and/or node and with the specialized version of its father (i.e., the version of the literal that originated the call) and carries around a list with the nodes on which the current goal depends. The analysis starts with a query (a goal) and a call substitution. With this information, `call_to_success` creates the root node of the tree and the list of clauses that unify with the goal. If the goal corresponds to a non-recursive predicate, it computes the success substitution which is asserted in a memo-table to reuse the result later on. Otherwise, the goal corresponds to a recursive predicate and it is dealt with by the fixpoint algorithm: first, it evaluates the non-recursive clauses obtaining an approximation of the success substitution and, after this, it starts the fixpoint computation.

During the fixpoint computation, for a goal with a given call substitution:

- If complete information has been already inferred and saved, `call_to_success` reuses it, to avoid re-computations.

¹ While it is true that abstract domains can be infinite, if convergence is not reached after some time, a widening operation changes the representation of the abstract substitutions to a coarser domain that has more chances to converge (or is sure to converge, if it is finite).

² The code is available at www.ciao-lang.org. For the reader convenience, we sketch it in Appendix B.

- If it is already inside a fixpoint computation (some parent started a fixpoint with the same call), `call_to_success` reuses the approximation stored for this call, to avoid entering loops.
- If an analyzed call depends on other nodes whose fixpoint are not completed yet, two cases are treated:
 - If the information on which the predicate depends is updated, a local fixpoint computation is started.
 - Otherwise, nothing is done.

To decide whether updated information for a node is available, the information inferred for it has a version number:

- When the information on a node is updated, its version number is increased by one.
- When a node uses information from another node, it stores the version of that information in the list of nodes on which it depends.

Version numbers are used to detect updates of the information on which a node analysis depend. If the version number of the last information used from a node does not match its current version number, there has been an update that needs to be propagated.

When the fixpoint computation finishes and the list of dependent nodes is empty, the current information for this call is asserted. Otherwise, if this list is not empty, the information remains flagged as an approximation and the fixpoint restarts. As it can easily be seen, while the algorithm can be conceptually not too complex, its implementation is cumbersome and at points costly, since many interactions are done through the database using identifiers for program points.

4.2 The PLAI Algorithm in TCLP

The PLAI code using tabling is a simplification of the corresponding Prolog implementation. The main points that were changed are:

- The handling of dependencies among nodes and the detection of termination in the fixpoint computation, that were explicit in the Prolog version, are now transferred to the underlying fixpoint of the tabling engine.
- The calculation of the LUB of the abstract substitutions generated by different clauses unifying with a call is done via lattice-based constraint aggregation (which is in turn built upon tabling).

4.2.1 Internal Database and Dependencies

In the Prolog implementation, the information related to the abstract substitutions is kept in a dynamic database relating code, program points, entry/exit substitutions, and dependencies. This makes it globally accessible and allows it to survive across backtracking and calls, so that it does not need to be carried around the program and be rebuilt every time there is a change in the substitution at a program point.

However, making the abstract interpreter update that information, switch among calls, and re-analyze calls needs accessing and updating this database, which is costly and mixes declarative and imperative styles. On top of that, the CiaoPP implementation has been fine-tuned during many years to avoid unnecessary (re-)analyses and minimize the overhead of accessing the

```

1 call_to_success(SgKey,Call,Proj,Sg,Sv,AbsInt,Succ) :-
2     call_to_success_fixpoint(SgKey,Sg, st(Sv,Call,Proj,AbsInt,Prime) ),
3     each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
4
5 :- use_package(tclp_aggregate).
6 :- table call_to_success_fixpoint(,_,abst_lub).
7 call_to_success_fixpoint(SgKey,Sg, st(Sv,Call,Proj,AbsInt,Prime) ) :-
8     trans_clause(SgKey,_,Clause),
9     do_nr_cl(Clause,Sg,Sv,Call,Proj,AbsInt,Prime).
10 call_to_success_fixpoint(SgKey,Sg, st(Sv,_Call,Proj,AbsInt,Prime) ) :-
11     \+ trans_clause(SgKey,_,_),
12     apply_trusted0(Proj,SgKey,Sg,Sv,AbsInt,_ClId,Prime).

```

Fig. 2: Implementation of `call_to_success/7` under the TCLP framework

database. All of these optimizations cause the code to have to deal with specific cases for the sake of performance, hence adding to its complexity. But despite the involved implementation, this machinery mimics, at Prolog level, an infrastructure similar to a tabling engine, but specialized for a given program—the abstract interpreter—and with optimizations specific for the task at hand.

This bookkeeping becomes unnecessary when using a tabling-based implementation. An abstract interpreter written using tabling and equipped with the capability to detect when two syntactically different substitutions represent the same object, can automatically take care of termination, suspend analysis when repeated calls are detected, and resume them when new information is available—all of it as part of the normal execution of a tabled program, without having to explicitly update and check dependencies.

That makes the code much simpler (no dependencies, lists of pending goals, resuming, etc. need to be explicitly coded) and shorter (we have obtained a threefold reduction in code size). On the other hand, the tabling engine is generic and cannot decide which suspension and/or resumption policy is better for a particular application. We on purpose chose to (a) keep the TCLP code simple and not include any specific heuristic in the code, (b) not to reimplement an analyzer from scratch, but simplify existing code, and (c) keep exactly the same interfaces (both those offered to the rest of CiaoPP and those required by the fixpoint code) so that the TCLP-based abstract interpreter can interoperate with the rest of the CiaoPP machinery as a drop-in replacement with close to zero effort. For these and other reasons, our performance figures (Section 5) are a lower bound of what could be achieved.

As an example, the implementation of `call_to_success/13` in Prolog checks several cases: if the call being analyzed is complete, under evaluation in a fixpoint, a call to a recursive predicate, a call to a non-recursive predicate, etc. to update information accordingly. It eventually invokes `proj_to_prime_nr/9`, which starts the fixpoint computation itself, and which recursively calls `call_to_success/13`. `call_to_success/13` has eight clauses and `proj_to_prime_nr/9` has six clauses (see Appendix B or the corresponding file at <http://www.cliplab.org/papers/tclp-plai-iclp2019>).

In the tabling implementation, the underlying engine and the calls to the abstract domain operations through the constraint solver interface take care of these cases and dependencies. This makes the implementation of `call_to_success` have just **one** clause (Fig. 2). The counterpart

```

1 call_entail(abst_lub, st(Sv,_,ProjA,AbsInt,_), st(Sv,_,ProjB,AbsInt,_)) :-
2     identical_abstract(AbsInt,ProjA,ProjB).
3 answer_entail(abst_lub, st(Sv,_,_,AbsInt,PrimeA), st(Sv,_,_,AbsInt,PrimeB)) :-
4     less_or_equal(AbsInt,PrimeA,PrimeB).
5 answer_join(abst_lub,st(Sv,_,_,Abs, A), st(Sv,_,_,Abs, B), st(Sv,_,_,Abs,New)) :-
6     compute_lub(Abs,[A,B],New).
7 apply_answer(abst_lub, st(Sv,_,_,AbsInt,Prime), st(Sv,_,_,AbsInt,Prime)).

```

Fig. 3: Code of the operator `abst_lub` under the TCLP framework

to `proj_to_prime_nr/9` (which we renamed `call_to_success_fixpoint/3` for clarity) has just two clauses: one for user predicates and another one for library and builtin predicates.

Additionally, the use of tabling makes it unnecessary to save explicitly all the intermediate substitutions, database identifiers for calls and program points, dependencies among goals, etc. This reduces the number of arguments, and `call_to_success` went from thirteen used in Prolog:

```
call_to_success(RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,ClId,Succ,List,F,N,Id)
```

to seven in the tabling-based implementation:

```
call_to_success(SgKey,Call,Proj,Sg,Sv,AbsInt,Succ)
```

4.2.2 Deciding Termination and Computing the LUB

In the PLAI algorithm, the different exit substitutions obtained from the clauses that unify with a given call are combined using the LUB operator of the abstract domain (Algorithm 2): exit substitutions $\beta_{i\text{exit}}$, for every clause C_i are joined to return the success substitution λ_{success} .

The CiaoPP implementation uses `bagof/3` to collect all the clauses in a list and then traverses it and analyzes every clause to create another list of abstract substitutions that are joined with the LUB. This processing is conceptually simple, but its implementation obscures the code with low-level operations, does not match the idea of having an interpreter executing on an abstract domain, and requires database accesses to retrieve the substitution applicable at that point.

In our implementation, the use of lattice-based aggregates with the tabling engine (Arias and Carro 2019b) simplifies the code. The `abst_lub` identifier in line 6 of Fig. 2 is the name of an interface that has several missions: determine suspension of calls, detect termination of the fixpoint, and perform aggregation of abstract substitutions. In the same line, the underscores state that the corresponding arguments are to be checked for equality (necessary to decide whether a fixpoint has been reached) using the *variant* policy, i.e., syntactical equality modulo variable renaming.

The implementation of the interface named `abst_lub` in Fig. 3 tells the tabling engine how to treat the argument selected previously with this identifier. In particular, the tabling engine checks the corresponding arguments for equality by calling `call_entail/3`. In our case, two abstract substitutions are termed equal if the abstract domain implementation (`identical_abstract/3`) decides so. This makes it possible to detect that two different representations correspond to the same object in the lattice and, if so, suspend a call or retrieve saved answers for it.

The code in Fig. 3 also aggregates the results returned in the third argument (the abstract substitutions) by joining them with the LUB of the lattice. The tabling engine calls `answer_entail/3` to decide whether a new answer (a substitution) is or not more general than an existing an-

swer (`less_or_equal/3`). If its not comparable, `answer_join/4` (which in turn invokes `compute_lub/3`) is called to compute the LUB of a previous answer and the new one. With these definitions, lines 7 to 12 in Fig. 2 contain **all** the code necessary to return the exit substitution of a call w.r.t. all its matching clauses. The implementation of the LUB operation (`abs_lub`, Fig. 3) is based on the operations provided by the abstract domain implementation.

This code also performs an incremental computation of the LUB as follows: upon success, the first answer, corresponding to the exit substitution $\beta_{1_{exit}}$, is stored in the answer table of the tabled predicate. Let us call this stored answer β_{exit} . For the subsequent exit substitutions $\beta_{i_{exit}}, i > 1$, there are two possible cases: if the saved substitution is more general ($\beta_{i_{exit}} \sqsubseteq \beta_{exit}$), then $\beta_{i_{exit}}$ is discarded; otherwise we make $\beta_{exit} = \beta_{exit} \sqcup \beta_{i_{exit}}$.

4.2.3 Connecting Abstract Substitutions with Lattice-Based Aggregates

The TCLP system handles entailment, aggregation, etc. by delegating operations to an underlying constraint solver using a fixed interface (Arias and Carro 2019a). Since we purposely did not change the representation of the CiaoPP abstract domains (they are used in other parts of the system), we constructed a bridge between these domains and the interface that TCLP expects.

The original entry point of the fixpoint, `proj_to_prime_nr/9` (renamed as `call_to_success_fixpoint/3` in the TCLP implementation), now tabled, is automatically rewritten (by the package `tclp_aggregate`) to call an auxiliary predicate that, at run time, substitutes the arguments carrying abstract substitutions by attributed variables (Holzbaur 1992) that simulate having a constrained variable. Their attributes are tuples that contain (a) the identifier (`abst_lub`, in our example) that determines the interface to be used and (b) the abstract substitution and ancillary information necessary by the abstract interpreter.

When one operation of the tabling engine involves a call with attributed variables, the engine checks if it has an attribute with contents it recognizes. If so, it calls the corresponding predicate from the interface that, in our case, operates on the substitution stored in the attributes.

5 Evaluation

Besides simplifying code, the implementation of PLAI using TCLP gives performance advantages in many cases. These come mainly because part of the bookkeeping related to dependencies, saving the analysis state when restarting the analysis of a dependent call, checking for termination, etc. are handled at a lower level. On the other hand, the implementation currently in CiaoPP, as commented before, has been fine-tuned and specialized during many years to minimize the overhead of the fixpoint implementation, so that a large proportion of the analysis time is spent in domain-related operations. On top of that, the CiaoPP domain representation and domain operations are designed to work well with its current architecture and coding decisions (e.g. saving and retrieving from the dynamic databases) and are suboptimal for a tabling-based implementation: for example, redundant data is manipulated and/or stored. As commented earlier, we did not change any of these so the TCLP fixpoint can seamlessly interact with the rest of the CiaoPP tool, exposing and using exactly the same interfaces.

Even with these constraints, we observed speedups when analyzing most programs from a benchmark set. We used the *Groundness* and *Sharing+Freeness* (Muthukumar and Hermenegildo 1991) domains due to their relevance (e.g., for program optimization and correctness of parallelization). *Groundness* (see Table 1 for performance results) determines if some program vari-

	Speedup	TCLP (ms)	CiaoPP (ms)
fibf_alt	1.60	0.29	0.46
aiakl	1.56	2.45	3.82
boyer	1.50	7.31	10.97
pv_queen	1.46	0.74	1.07
subst	1.41	0.25	0.35
pv_gabriel	1.37	3.65	4.99
rdtok	1.32	7.03	9.25
mmatf	1.24	0.31	0.39
hanoi	1.22	0.53	0.65
revf_lin	1.20	0.27	0.32
append	1.20	0.17	0.20
rev_lin	1.19	0.26	0.31
prefix	1.16	0.27	0.31
revf	1.15	0.32	0.37
pv_plan	1.15	1.94	2.23
sublist_app	1.14	0.24	0.27
reverse	1.14	0.38	0.43
flatten	1.13	0.55	0.62
palindro	1.12	0.34	0.38
fact	1.08	0.25	0.27
rotate	1.06	0.46	0.49
maxtree	0.98	0.63	0.61
zebra	0.92	1.38	1.26
browse	0.89	1.76	1.57
AVG	1.31	31.78	41.59

Table 1: Performance comparison: CiaoPP fixpoint in Prolog and TCLP (*Groundness* domain).

able will be bound to a ground term. This is useful to derive modes, optimize unification, and improve the precision of the *Sharing+Freeness* analysis, among others.

Sharing+Freeness (see Table 2) determines if two (or more) program variables may be bound to terms sharing a common variable. It is useful to determine, for example, whether running two goals in parallel may try to bind the same variable, thus causing races and compromising correctness. The benchmarks used are standard programs that have been previously used to evaluate CiaoPP.

All the experiments in this paper were performed on a Linux 5.0.0-13-generic machine with an Intel Core i7 at 1.80GHz with 16Gb of memory and using gcc 8.3.0 to compile the abstract machine of Ciao Prolog. In all cases, every program was analyzed 40 times and the 10 worst times were discarded, both when using the tabling and the Prolog implementation, to try to minimize the effect of spurious interruptions, O.S. scheduling, etc. that can introduce noise in the execution. The remaining times were averaged. All the code and the system under evaluation is available at <http://www.cliplab.org/papers/tclp-plai-iclp2019>.

The average speedups in each table were calculated by adding up the (averaged) execution times for all the benchmarks and dividing the *CiaoPP* time by the *TCLP* time. This shows that, on average, the analysis with the *Groundness* domain speeds up a bit more than 30%, while the analysis with the *Sharing+Freeness* has experienced, on average, a slight slowdown (about 3%).

By looking at every benchmark in isolation, we can observe that the speedups differ greatly

	Speedup	TCLP (ms)	CiaoPP (ms)
fact	1.30	0.26	0.33
pv_queen	1.23	1.21	1.49
mmatf	1.17	0.51	0.60
mmatrix	1.15	0.53	0.61
prefix	1.14	0.46	0.52
revf	1.12	0.47	0.53
revf_lin	1.10	0.39	0.43
reverse	1.10	0.39	0.43
rev_lin	1.10	0.38	0.42
rotate	1.06	0.72	0.76
pv_pg	1.01	2.67	2.70
append	0.98	1.11	1.09
sublist_app	0.96	0.87	0.84
zebra	0.91	16.34	14.80
AVG	0.97	26.31	25.55

Table 2: Performance comparison: CiaoPP fixpoint in Prolog and TCLP (*Sh+Fr* domain).

among them. We have sorted the benchmarks according to the speedup to appreciate better the differences. In both cases, only a small part of the benchmarks (three) experienced a slowdown, and even in these cases, the maximum slowdown was about 10%. In the case of *Sharing+Freeness*, the slowest analysis corresponded as well to the largest execution time (larger than the rest of the benchmarks combined). We want to note that this benchmark (zebra) is probably not a representative of a typical program, as it is a combinatorial problem with many free variables in a single clause, some of which are aliased with each other.

The source of the speed difference is not easy to determine. A profile of the number of fixpoint calls in CiaoPP vs. fixpoint calls, entailment checks, joins, etc. in the TCLP version does not seem to show a correlation with the observed speedups. We therefore conjecture that the shape and size of the abstract substitution, and the relative cost of checking entailment, has to be explored to have a better explanation of the differences observed.

6 Conclusions and Future Work

We have presented a re-implementation of PLAI, a fixpoint computation algorithm for abstract interpretation, using tabled constraint logic programming. The resulting code is considerably shorter than the current Prolog implementation of PLAI in CiaoPP (one-third of its size) and much simpler: all the bookkeeping necessary to keep track of dependencies between predicates, analysis restarting, etc. is in charge of the tabling engine, which increases the maintainability of the implementation of PLAI.

We have evaluated its performance using several benchmarks and abstract domains, and compared it with the original implementation in CiaoPP. In most cases, the TCLP implementation showed improved performance, sometimes with a speedup of 60%. In a few cases there was a small slowdown, which we think is a reasonable price to pay for the added code clarity, especially taking into account that there is room for improvement in the current implementation.

Among the immediate future plans, we want to experiment re-implementing the abstract domains with an optimized representation of the abstract substitutions, and also use constraint logic

programming techniques to propagate the effects of updates. We also expect that, using constraints, we will be able to define widening heuristics independently of the fixpoint algorithm thereby increasing the resulting flexibility, precision and performance w.r.t. the state of the art.

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Appendix A PLAI Algorithm Implementation Using TCLP

In this appendix we include the code corresponding to the reimplementaion of PLAI using TCLP. It is not expected to be used to understand the code (we did not add any facility or improve its functionality), but rather to compare the code length and complexity with that of the original PLAI in CiaoPP, which we include in Appendix B. Therefore, we have removed the comments that appear in the original files. The files with comments can be accessed at <http://www.cliplab.org/papers/tclp-plai-iclp2019>.

```

1  /*          Copyright (C)1990-2019 UPM-CLIP          */
2
3  :- module(fixpo_plai_tabling,
4          [
5          query/8,
6          init_fixpoint/0,
7          cleanup_fixpoint/1,
8          entry_to_exit/9
9          ],
10         [assertions, datafacts]).
11
12 % Ciao library
13 :- use_module(engine(io_basic)).
14
15 :- use_module(library(agggregates), [bagof/3, (^)/2]).
16 :- use_module(library(lists), [member/2, append/3]).
17 :- use_module(library(terms_vars), [varset/2]).
18 :- use_module(library(terms_check)).
19 :- use_module(library(sets), [merge/3, ord_subtract/3]).
20 :- use_module(library(sort), [sort/2]).
21 :- use_module(library(messages)).
22 :- use_module(library(write)).
23
24 % CiaoPP library
25 :- use_module(ciaopp(preprocess_flags), [current_pp_flag/2, set_pp_flag/2]).
26
27 % Plai library
28 :- use_module(ciaopp(plai/fixpo_ops), [inexistent/2, variable/2, bottom/1,
29         singleton/2, fixpoint_id_reuse_prev/5, fixpoint_id/1, fixp_id/1,
30         each_abs_sort/3,
31         % each_concrete/4,
32         each_extend/6, each_project/6, each_exit_to_prime/8, each_unknown_call/4,
33         each_body_succ_builtin/12, body_succ_meta/7, reduce_equivalent/3,
34         each_apply_trusted/7, widen_succ/4, decide_memo/6, clause_applies/2,
35         abs_subset_/3]).
36
37 :- use_module(ciaopp(plai/domains)).
38 :- use_module(ciaopp(plai/trace_fixp), [fixpoint_trace/7, cleanup/0]).
39 :- use_module(ciaopp(plai/plai_db),
40         [ complete/7, memo_call/5, memo_table/6, cleanup_plai_db/1, patch_parents/6 ]).
41 :- use_module(ciaopp(plai/psets), [update_if_member_idlist/3]).
42 :- use_module(ciaopp(plai/re_analysis), [erase_previous_memo_tables_and_parents/4]).
43 :- use_module(ciaopp(plai/transform), [body_info/4, trans_clause/3]).
44 :- use_module(ciaopp(plai/apply_assertions_old),
45         [ apply_trusted/7,
46         cleanup_trusts/1 ]).
47
48 :- doc(author, "Joaquin Arias").
49
50 :- doc(module, "This module adapts the implementation of the top-down
51         fixpoint algorithm of PLAI using TCLP with aggregates and an
52         extension that also checks call entailment.").
53
54 init_fixpoint.

```

```

55
56 cleanup_fixpoint(_AbsInt).
57
58 %------%
59 % call_to_success(+,+,+,+,+,-) %
60 %------%
61
62 call_to_success(SgKey,Call,Proj,Sg,Sv,AbsInt,Succ) :-
63     call_to_success_fixpoint(SgKey,Sg, st(Sv,Call,Proj,AbsInt,Prime)),
64     each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
65
66 %%%%%%%%%% TCLP interface %%%%%%%%%%
67 :- use_package(tclp_aggregate).
68 :- table call_to_success_fixpoint(_,_ , abst_lub).
69
70 call_entail(abst_lub, st(V,_ ,ProjA,AbsInt,_), st(V,_ ,ProjB,AbsInt,_)) :-
71     identical_abstract(AbsInt,ProjA,ProjB), !.
72
73 answer_entail(abst_lub, st(V,_ ,_ ,AbsInt,PrimeAs), st(V,_ ,_ ,AbsInt,PrimeBs),1) :-
74     singleton(PrimeA,PrimeAs),
75     singleton(PrimeB,PrimeBs),
76     less_or_equal(AbsInt,PrimeA,PrimeB), !.
77
78 answer_join(abst_lub,st(V,_ ,_ ,AbsInt,PrimeAs), st(V,_ ,_ ,AbsInt,PrimeBs),
79             st(V,_ ,_ ,AbsInt,PrimeNews)) :-
80     singleton(PrimeA,PrimeAs),
81     singleton(PrimeB,PrimeBs),
82     singleton(PrimeNew,PrimeNews),
83     compute_lub(AbsInt,[PrimeA,PrimeB],PrimeNew), !.
84
85 apply_answer(abst_lub, st(V,_ ,_ ,Ab,A), st(V,_ ,_ ,Ab,B)) :- A = B.
86
87 call_to_success_fixpoint(SgKey,Sg,st(Sv,Call,Proj,AbsInt,Primes)) :-
88     trans_clause(SgKey,_ ,Clause),
89     do_nr_cl(Clause,Sg,Sv,Call,Proj,AbsInt,Primes).
90 call_to_success_fixpoint(SgKey,Sg,st(Sv,_Call,Proj,AbsInt,Primes)) :-
91     \+ trans_clause(SgKey,_ ,_),
92     apply_trusted0(Proj,SgKey,Sg,Sv,AbsInt,_ClId,Prime),
93     singleton(Prime,Primes).
94
95 do_nr_cl(Clause,Sg,Sv,Call,Proj,AbsInt,Primes):-
96     Clause = clause(Head,Vars_u,K,Body),
97     clause_applies(Head,Sg), !,
98     varset(Head,Hv),
99     sort(Vars_u,Vars),
100    ord_subtract(Vars,Hv,Fv),
101    process_body(Body,K,AbsInt,Sg,Hv,Fv,Vars_u,Head,Sv,Call,
102                Proj,Primes,_Id).
103 do_nr_cl(_Clause,_Sg,_Sv,_Call,_Proj,_AbsInt,[[ ]]).
104
105 process_body(Body,K,AbsInt,Sg,Hv,_Fv,_ ,Head,Sv,Call,Proj,LPrime,_Id):-
106     Body = g(_ ,[[,'$built'(_ ,true,_),'true/0',true]), !,
107     singleton(Prime,LPrime),
108     call_to_success_fact(AbsInt,Sg,Hv,Head,K,Sv,Call,Proj,Prime,_Succ).
109 process_body(Body,K,AbsInt,Sg,Hv,Fv,Vars_u,Head,Sv,_ ,Proj,Prime,Id):-
110     call_to_entry(AbsInt,Sv,Sg,Hv,Head,K,Fv,Proj,Entry,ExtraInfo),
111     singleton(Entry,LEntry),
112     entry_to_exit(Body,K,LEntry,Exit,[_ ,_ ,Vars_u,AbsInt,Id],
113     each_exit_to_prime(Exit,AbsInt,Sg,Hv,Head,Sv,ExtraInfo,Prime).
114
115 %------%
116 % entry_to_exit(+,+,+,-,+,-,+ ,+) %
117 %------%
118
119 entry_to_exit((Sg,Rest),K,Call,Exit,OldList,NewList,Vars_u,AbsInt,NewN):- !,
120     body_succ(Call,Sg,Succ,OldList,IntList,Vars_u,AbsInt,K,NewN,_),

```

```

121     entry_to_exit(Rest,K,Succ,Exit,IntList,NewList,Vars_u,AbsInt,NewN) .
122 entry_to_exit(true,_,Call,Call,List,List,_,_,_):- !.
123 entry_to_exit(Sg,Key,Call,Exit,OldList,NewList,Vars_u,AbsInt,NewN):-
124     body_succ(Call,Sg,Exit,OldList,NewList,Vars_u,AbsInt,Key,NewN,_),
125     true.
126
127 body_succ(Call,_Atom,Succ,List,List,_HvFv_u,_AbsInt,_CId,_ParentId,no):-
128     bottom(Call), !,
129     Succ = Call.
130 body_succ(Call,Atom,Succ,List,NewList,HvFv_u,AbsInt,CId,ParentId,Id):-
131     Atom=g(Key,Sv,Info,SgKey,Sg),
132     body_succ_(Info,SgKey,Sg,Sv,HvFv_u,Call,Succ,List,NewList,AbsInt,
133     CId,Key,ParentId,Id) .
134
135 body_succ_(Info,SgKey,Sg,Sv,HFv,Call,Succ,L,NewL,AbsInt,CId,Key,PIId,Id):-
136     Info = [_|_], !,
137     split_combined_domain(AbsInt,Call,Calls,Domains),
138     map_body_succ(Info,SgKey,Sg,Sv,HFv,Calls,Succs,L,NewL,Domains,
139     CId,Key,PIId,Id),
140     split_combined_domain(AbsInt,Succ,Succs,Domains) .
141 body_succ_(Info,SgKey,Sg,Sv,HFv,Call,Succ,L,NewL,AbsInt,CId,Key,PIId,Id):-
142     body_succ0(Info,SgKey,Sg,Sv,HFv,Call,Succ,L,NewL,AbsInt,
143     CId,Key,PIId,Id) .
144
145 map_body_succ([],_SgKey,_Sg,_Sv,_HFv,[],[],L,L,[],_CId,_Key,_PIId,no) .
146 map_body_succ([I|Info],SgKey,Sg,Sv,HFv,[Call|Calls],[Succ|Succs],L,NewL,
147     [AbsInt|Domains],CId,Key,PIId,Id):-
148     body_succ0(I,SgKey,Sg,Sv,HFv,Call,Succ,L,_NewL,AbsInt,
149     CId,Key,PIId,_Id), !,
150     map_body_succ(Info,SgKey,Sg,Sv,HFv,Calls,Succs,L,NewL,Domains,
151     CId,Key,PIId,Id) .
152
153 body_succ0('$var',SgKey,Sg,_Sv_u,HvFv_u,Calls,Succs,List0,List,AbsInt,
154     _CId,F,_N,_Id):-
155     !,
156     ( Calls=[Call],
157     concrete(AbsInt,Sg,Call,Concretes),
158     concretes_to_body(Concretes,SgKey,AbsInt,B)
159     -> meta_call(B,HvFv_u,Calls,[],Succs,List0,List,AbsInt,_CId,_Id,_Ids)
160     ; List=List0,
161     each_unknown_call(Calls,AbsInt,[Sg],Succs) % Sg is a variable
162     ) .
163 body_succ0('$meta'(T,B,_),SgKey,Sg,Sv_u,HvFv_u,Call,Succ,List0,List,AbsInt,
164     _CId,_F,_N,_Id):-
165     !,
166     meta_call(B,HvFv_u,Call,[],Exits,List0,List,AbsInt,CId,Id,_Ids),
167     ( body_succ_meta(T,AbsInt,Sv_u,HvFv_u,Call,Exits,Succ) ->
168     true
169     ; % for the trusts, if any:
170     varset(Sg,Sv_r), % Sv_u contains extra vars (from meta-term)
171     % which will confuse apply_trusted
172     body_succ0(nr,SgKey,Sg,Sv_r,HvFv_u,Call,Succ,[],_List,AbsInt,
173     _CId,_F,_N,_Id0)
174     ) .
175 body_succ0('$built'(T,Tg,Vs),SgKey,Sg,Sv_u,HvFv_u,Call,Succ,List0,List,AbsInt,
176     _CId,_F,_N,_Id):-
177     !,
178     List=List0,
179     sort(Sv_u,Sv),
180     each_body_succ_builtin(Call,AbsInt,T,Tg,Vs,SgKey,Sg,Sv,HvFv_u,Succ) .
181 body_succ0(_RFlag,SgKey,Sg,Sv_u,HvFv_u,Call,Succ,_List0,_List,AbsInt,
182     _CId,_F,_N,_Id):-
183     sort(Sv_u,Sv),
184     each_call_to_success(Call,SgKey,Sg,Sv,HvFv_u,AbsInt,Succ) .
185
186 %% predicate adapted from fixpo_ops

```

```

187 each_body_succ_builtin_([],_,_T,_Tg,_,_,_Sg,_Sv,_HvFv_u, []).
188 each_body_succ_builtin_([Call|Calls],AbsInt,T,Tg,Vs,SgKey,Sg,Sv,HvFv_u,[Succ|Succs]):-
189     project(AbsInt,Sg,Sv,HvFv_u,Call,Proj),
190     body_succ_builtin(T,AbsInt,Tg,Vs,Sv,HvFv_u,Call,Proj,Succ),!, %% Doamin call
191     each_body_succ_builtin_tabling_(Calls,AbsInt,T,Tg,Vs,SgKey,Sg,Sv,HvFv_u,Succs).
192
193 each_call_to_success([Call],SgKey,Sg,Sv,HvFv_u,AbsInt,Succ):-
194     !,
195     project(AbsInt,Sg,Sv,HvFv_u,Call,Proj),
196     call_to_success(SgKey,Call,Proj,Sg,Sv,AbsInt,Succ).
197
198 each_call_to_success(LCall,SgKey,Sg,Sv,HvFv_u,AbsInt,LSucc):-
199     each_call_to_success0(LCall,SgKey,Sg,Sv,HvFv_u,AbsInt,
200     LSucc).
201
202 each_call_to_success0([],_SgK,_Sg,_Sv,_HvFv_u,AbsInt, []).
203 each_call_to_success0([Call|LCall],SgKey,Sg,Sv,HvFv_u,AbsInt,
204     LSucc):-
205     project(AbsInt,Sg,Sv,HvFv_u,Call,Proj),
206     call_to_success(SgKey,Call,Proj,Sg,Sv,AbsInt,LSucc0),
207     append(LSucc0,LSucc1,LSucc),
208     each_call_to_success0(LCall,SgKey,Sg,Sv,HvFv_u,AbsInt,
209     LSucc1).
210
211 meta_call([],_HvFv_u,Call,[],Call,List,List,_AbsInt,_CId,_Id, []).
212 meta_call([Body|Bodies],HvFv_u,Call,Succ0,Succ,L0,List,AbsInt,CId,Id,Ids):-
213     meta_call_([Body|Bodies],HvFv_u,Call,Succ0,Succ,L0,List,AbsInt,CId,Id,Ids).
214 meta_call_([Body|Bodies],HvFv_u,Call,Succ0,Succ,L0,List,AbsInt,CId,Id,Ids):-
215     meta_call_body(Body,CId,Call,Succ1,L0,L1,HvFv_u,AbsInt,Id,Ids0),
216     widen_succ(AbsInt,Succ0,Succ1,Succ2),
217     append(Succ0,Succ1,Succ2),
218     append(Ids0,Ids1,Ids),
219     meta_call_(Bodies,HvFv_u,Call,Succ2,Succ,L1,List,AbsInt,CId,Id,Ids1).
220 meta_call_([],_HvFv_u,_Call,Succ,Succ,List,List,_AbsInt,_CId,_Id, []).
221
222 meta_call_body((Sg,Rest),K,Call,Exit,OldList,NewList,Vars_u,AbsInt,PId,CIds):-
223     !,
224     CIds=[Id|Ids],
225     body_succ(Call,Sg,Succ,OldList,IntList,Vars_u,AbsInt,K,PId,Id),
226     meta_call_body(Rest,K,Succ,Exit,IntList,NewList,Vars_u,AbsInt,PId,Ids).
227 meta_call_body(true,_,Call,Call,List,List,_,_,_,[no]):-!.
228 meta_call_body(Sg,Key,Call,Exit,OldList,NewList,Vars_u,AbsInt,PId,[Id]):-
229     body_succ(Call,Sg,Exit,OldList,NewList,Vars_u,AbsInt,Key,PId,Id).
230
231 concretes_to_body([],_SgKey,_AbsInt, []).
232 concretes_to_body([Sg|Sgs],SgKey,AbsInt,[B|Bs]):-
233     body_info0(Sg:SgKey,[],AbsInt,B),
234     concretes_to_body(Sgs,SgKey,AbsInt,Bs).
235
236 %------%
237 % query(+,+,+,+,+,+,+,+) %
238 %------%
239
240 :- doc(query(AbsInt,QKey,Query,Qv,RFlag,N,Call,Succ),
241     "The success pattern of @var{Query} with @var{Call} is
242     @var{Succ} in the analysis domain @var{AbsInt}. The predicate
243     called is identified by @var{QKey}. The goal @var{Query} has
244     variables @var{Qv}.").
245
246 query(AbsInt,QKey,Query,Qv,_RFlag,_N,Call,Succ) :-
247     project(AbsInt,Query,Qv,Qv,Call,Proj),
248     call_to_success(QKey,Call,Proj,Query,Qv,AbsInt,Succ), !.
249
250 query(_AbsInt,_QKey,_Query,_Qv,_RFlag,_N,_Call,_Succ):-
251     % should never happen, but...
252     error_message("SOMETHING HAS FAILED!").

```

Appendix B PLAI Algorithm Implementation in Ciao Prolog

We include here the Ciao Prolog implementation of PLAI. As mentioned before, we have removed the comments from the file since the goal of this appendix it to make it easier for the reader to compare the Ciao Prolog code w.r.t. the code using TCLP, which we include in Appendix A. The original version is available at <http://www.cliplab.org/papers/tclp-plai-iclp2019>.

```

1  /*          Copyright (C)1990-2019 UPM-CLIP          */
2
3  :- module(fixpo_plai_with_comments,
4     [ query/8,
5       init_fixpoint/0,
6       cleanup_fixpoint/1,
7       entry_to_exit/9
8     ],
9     [assertions, datafacts]).
10
11 % Ciao library
12 :- use_module(library(agggregates), [bagof/3, (~)/2]).
13 :- use_module(library(lists), [member/2, append/3]).
14 :- use_module(library(terms_vars), [varset/2]).
15 :- use_module(library(sets), [merge/3, ord_subtract/3]).
16 :- use_module(library(sort), [sort/2]).
17 :- use_module(library(messages)).
18
19 % CiaoPP library
20 :- use_module(ciaopp(preprocess_flags), [current_pp_flag/2, set_pp_flag/2]).
21
22 % Plai library
23 :- use_module(ciaopp(plai/fixpo_ops), [inexistent/2, variable/2, bottom/1,
24   singleton/2, fixpoint_id_reuse_prev/5, fixpoint_id/1, fixp_id/1,
25   each_abs_sort/3,
26   each_extend/6, each_project/6, each_exit_to_prime/8, each_unknown_call/4,
27   each_body_succ_builtin/12, body_succ_meta/7, reduce_equivalent/3,
28   each_apply_trusted/7, widen_succ/4, decide_memo/6, clause_applies/2,
29   abs_subset_/3]).
30
31 :- use_module(ciaopp(plai/domains)).
32 :- use_module(ciaopp(plai/trace_fixp), [fixpoint_trace/7, cleanup/0]).
33 :- use_module(ciaopp(plai/plai_db),
34   [ complete/7, memo_call/5, memo_table/6, cleanup_plai_db/1, patch_parents/6 ]).
35 :- use_module(ciaopp(plai/psets), [update_if_member_idlist/3]).
36 :- use_module(ciaopp(plai/re_analysis), [erase_previous_memo_tables_and_parents/4]).
37 :- use_module(ciaopp(plai/transform), [body_info0/4, trans_clause/3]).
38 :- use_module(ciaopp(plai/apply_assertions_old),
39   [ apply_trusted0/7,
40     cleanup_trusts/1 ]).
41
42 :- doc(author, "Kalyan Muthukumar").
43 :- doc(author, "Maria Garcia de la Banda").
44 :- doc(author, "Francisco Bueno").
45
46 :- doc(module, "This module implements the top-down fixpoint
47   algorithm of PLAI, both in its mono-variant and multi-variant
48   on successes versions. It is always multi-variant on calls.
49   The algorithm is parametric on the particular analysis domain.").
50
51
52 :- data '$depend_list'/3.
53 :- data ch_id/2.
54
55 :- data approx/6.
56 :- data fixpoint/6.

```

```

57 :- data fixpoint_variant/6.
58 :- data approx_variant/7.
59
60 init_fixpoint:-
61     retractall_fact(approx(_,_,-,-,-,-)),
62     retractall_fact(fixpoint(_,-,-,-,-,-)),
63     retractall_fact('$depend_list'(_,-,-,-)),
64     retractall_fact(ch_id(_,-)),
65     retractall_fact(fixpoint_variant(_,-,-,-,-,-)),
66     retractall_fact(approx_variant(_,-,-,-,-,-)),
67     trace_fixp:cleanup.
68
69 cleanup_fixpoint(AbsInt):-
70     cleanup_plai_db(AbsInt),
71     cleanup_trusts(AbsInt),
72     retractall_fact(fixp_id(_)),
73     asserta_fact(fixp_id(0)), % there is no way to recover this
74     init_fixpoint.          % if several analyses coexist!
75
76 approx_to_completes(AbsInt):-
77     current_fact(approx(SgKey,Sg,Proj,Prime,Pid,Fs),Ref),
78     asserta_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Pid,Fs)),
79     erase(Ref),
80     fail.
81 approx_to_completes(AbsInt):-
82     current_fact(approx_variant(_Id,Pid,SgKey,Sg,Proj,Prime,Fs),Ref),
83     asserta_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Pid,Fs)),
84     erase(Ref),
85     fail.
86 approx_to_completes(_AbsInt).
87
88
89 %-----%
90 % call_to_success(+,+,+,+,+,+,+,-,+,,+) %
91 %-----%
92
93 call_to_success(_RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,_CId,Succ,List,F,N,Id):-
94     % CId = number identifying the clause?... for an entry point is 0...
95     % F = program point of the call. clauseId+0 for an entry call
96     current_fact(complete(SgKey,AbsInt,Subg,Proj1,Prime1,Id,Fs),R),
97     identical_proj(AbsInt,Sg,Proj,Subg,Proj1), !,
98     patch_parents(R,complete(SgKey,AbsInt,Subg,Proj1,Prime1,Id,Ps),F,N,Ps,Fs),
99     List = [],
100     each_abs_sort(Prime1,AbsInt,Prime),
101     each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
102 call_to_success(r,SgKey,Call,Proj,Sg,Sv,AbsInt,_CId,Succ,List,F,N,Id) :-
103     current_fact(approx(SgKey,Subg,Proj1,Prime1,Id,Fs),Ref),
104     identical_proj(AbsInt,Sg,Proj,Subg,Proj1), !,
105     each_abs_sort(Prime1,AbsInt,TempPrime),
106     current_fact('$depend_list'(Id,SgKey,IdList)),
107     call_to_success_approx(SgKey,Subg,Call,Proj,Proj1,Sg,Sv,AbsInt,F,N,Fs,
108         Id,Ref,IdList,Prime1,TempPrime,List,Prime),
109     each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
110 call_to_success(r,SgKey,Call,Proj,Sg,Sv,AbsInt,_CId,Succ,List,F,N,Id) :-
111     current_fact(fixpoint(SgKey,Subg,Proj1,Prime1,Id,Fs),Ref),
112     identical_proj(AbsInt,Sg,Proj,Subg,Proj1), !,
113     patch_parents(Ref,fixpoint(SgKey,Subg,Proj1,Prime1,Id,Ps),F,N,Ps,Fs),
114     current_fact(ch_id(Id,Num)),
115     List = [Id/Num],
116     each_abs_sort(Prime1,AbsInt,Prime),
117     each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
118 call_to_success(_RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,_CId,Succ,List,F,N,Id):-
119     current_pp_flag(variants,on),
120     current_fact(complete(SgKey,AbsInt,Subg,Proj1,Prime1,_Id1,_Fs),_R),
121     identical_proj_1(AbsInt,Sg,Proj,Subg,Proj1,Prime1,Prime2), !,
122     format("call to success tipe _RFlag SgKey",[]),

```

```

123     ( current_pp_flag(reuse_fixp_id,on) ->
124         fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
125     ;
126         fixpoint_id(Id)
127     ),
128     each_abs_sort(Prime2,AbsInt,Prime),
129     List = [],
130     asserta_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Id,[(F,N)])),
131     each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
132 call_to_success(r,SgKey,Call,Proj,Sg,Sv,AbsInt,_ClId,Succ,List,F,N,Id) :-
133     current_pp_flag(variants,on),
134     current_fact(approx(SgKey,Subg,Proj1,Prime1,Id1,Fs),Ref),
135     identical_proj_1(AbsInt,Sg,Proj,Subg,Proj1,Prime1,Prime2), !,
136     each_abs_sort(Prime2,AbsInt,TempPrime),
137     current_fact('$depend_list'(Id1,SgKey,IdList)),
138     call_to_success_approx_variant(SgKey,Subg,Call,Proj,Proj1,Sg,Sv,AbsInt,F,N,Fs,
139         Id,Id1,Ref,IdList,Prime1,TempPrime,List,Prime),
140     each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
141 call_to_success(r,SgKey,Call,Proj,Sg,Sv,AbsInt,_ClId,Succ,List,F,N,Id) :-
142     current_pp_flag(variants,on),
143     current_fact(fixpoint(SgKey,Subg,Proj1,Prime1,Id1,_Fs),_Ref),
144     identical_proj_1(AbsInt,Sg,Proj,Subg,Proj1,Prime1,Prime2), !,
145     (
146         current_fact(fixpoint_variant(Id1,Id,SgKey,Sgv,Projv,Fsv),Refv),
147         identical_proj(AbsInt,Sg,Proj,Sgv,Projv) ->
148         patch_parents(Refv,fixpoint_variant(Id1,Id,SgKey,Sgv,Projv,Ps),F,N,Ps,Fsv)
149     ;
150     (
151         current_pp_flag(reuse_fixp_id,on) ->
152         fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
153     ;
154         fixpoint_id(Id)
155     ),
156     asserta_fact(fixpoint_variant(Id1,Id,SgKey,Sg,Proj,[(F,N)]))
157     ),
158     each_abs_sort(Prime2,AbsInt,Prime),
159     current_fact(ch_id(Id1,Num)),
160     List = [Id1/Num],
161     each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
162 call_to_success(r,SgKey,Call,Proj,Sg,Sv,AbsInt,_ClId,Succ,List,F,N,Id) :-
163     init_fixpoint0(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,[(F,N)],Id,List,Prime),
164     each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
165 call_to_success(nr,SgKey,Call,Proj,Sg,Sv,AbsInt,ClId,Succ,[],F,N,Id) :-
166     ( current_pp_flag(reuse_fixp_id,on) ->
167         fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
168     ;
169         fixpoint_id(Id)
170     ),
171     proj_to_prime_nr(SgKey,Sg,Sv,Call,Proj,AbsInt,ClId,Prime,Id),
172     asserta_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Id,[(F,N)])),
173     each_extend(Sg,Prime,AbsInt,Sv,Call,Succ).
174
175 call_to_success_approx(SgKey,Subg,_Call,Proj,Proj1,Sg,_Sv,_AbsInt,F,N,Fs,
176     Id,Ref,IdList,Prime1,TempPrime,List,Prime):-
177     not_modified(IdList), !,
178     patch_parents(Ref,approx(SgKey,Subg,Proj1,Prime1,Id,Ps),F,N,Ps,Fs),
179     Prime = TempPrime,
180     List = IdList.
181 call_to_success_approx(SgKey,_Subg,Call,Proj,_Proj1,Sg,Sv,AbsInt,F,N,Fs,
182     Id,Ref,_IdList,_Prime1,TempPrime,List,Prime):-
183     erase(Ref),
184     init_fixpoint_(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Fs,Id,
185         TempPrime,List,Prime).
186
187 aproxs_to_fixpoint_variant(Id):-
188     current_fact(approx_variant(Id,Idv,SgKey,Sgv,Projv,_Primev,Fs),Ref),!,

```

```

189     erase(Ref),
190     asserta_fact(fixpoint_variant(Id,Idv,SgKey,Sgv,Projv,Fs)),
191     aproxs_to_fixpoint_variant(Id).
192 aproxs_to_fixpoint_variant(_).
193
194
195 call_to_success_approx_variant(SgKey,_Subg,_Call,Proj,_Proj1,Sg,_Sv,AbsInt,F,N,_Fs,
196                               Id,Id1,_Ref,IdList,_Prime1,TempPrime,List,Prime):-
197     not_modified(IdList),!,
198     (
199         current_fact(approx_variant(Id1,Id,SgKey,Sgv,Projv,Primev,Fsv),Refv),
200         identical_proj(AbsInt,Sg,Proj,Sgv,Projv) ->
201         patch_parents(Refv,approx_variant(Id1,Id,SgKey,Sgv,Projv,Primev,Ps),F,N,Ps,Fsv)
202     );
203     (
204         current_pp_flag(reuse_fixp_id,on) ->
205         fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
206     );
207     fixpoint_id(Id)
208     ),
209     asserta_fact(approx_variant(Id1,Id,SgKey,Sg,Proj,TempPrime,[F,N]))
210     ),
211     Prime = TempPrime,
212     List = IdList.
213 call_to_success_approx_variant(SgKey,Subg,Call,Proj,Proj1,Sg,Sv,AbsInt,F,N,Fs,
214                               Id,Id1,Ref,_IdList,Prime1,_TempPrime,List,Prime):-
215     (
216         current_fact(approx_variant(Id1,Id,SgKey,Sgv,Projv,_Primev,Fsv),Refv),
217         identical_proj(AbsInt,Sg,Proj,Sgv,Projv) ->
218         erase(Refv),
219         ( member((F,N),Fsv) -> NewFs = Fsv ; NewFs = [(F,N)|Fsv] %)
220     );
221     (
222         current_pp_flag(reuse_fixp_id,on) ->
223         fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
224     );
225     fixpoint_id(Id)
226     ),
227     NewFs = [(F,N)]
228     ),
229     aproxs_to_fixpoint_variant(Id1),
230     erase(Ref),
231     asserta_fact(fixpoint_variant(Id1,Id,SgKey,Sg,Proj,NewFs)),
232     varset(Subg,Subv),
233     init_fixpoint_(SgKey,Call,Proj1,Subg,Subv,AbsInt,F,N,Fs,Id1,
234                   Prime1,List,Prime0),
235     each_exit_to_prime(Prime0,AbsInt,Sg,Subv,Subg,Sv,(no,Proj),Prime).
236
237 init_fixpoint0(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Fs,Id,List,Prime):-
238     init_fixpoint2(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,Fs,Id,List,Prime).
239
240 init_fixpoint1(SgKey,_Call,Proj,Sg,_Sv,AbsInt,F,N,_Fs0,Id,List,Prime):-
241     current_fact(complete(SgKey,AbsInt,Subg,Proj1,Prime1,Id,Fs),R),
242     identical_proj(AbsInt,Sg,Proj,Subg,Proj1),!,
243     patch_parents(R,complete(SgKey,AbsInt,Subg,Proj1,Prime1,Id,Ps),F,N,Ps,Fs),
244     List = [],
245     each_abs_sort(Prime1,AbsInt,Prime).
246 init_fixpoint1(SgKey,Call,Proj,Sg,Sv,AbsInt,F,N,_Fs0,Id,List,Prime):-
247     current_fact(approx(SgKey,Subg,Proj1,Prime1,Id,Fs),Ref),
248     identical_proj(AbsInt,Sg,Proj,Subg,Proj1),!,
249     each_abs_sort(Prime1,AbsInt,TempPrime),
250     current_fact('$depend_list'(Id,SgKey,IdList)),
251     call_to_success_approx(SgKey,Subg,Call,Proj,Proj1,Sg,Sv,AbsInt,F,N,Fs,
252                           Id,Ref,IdList,Prime1,TempPrime,List,Prime).
253 init_fixpoint1(SgKey,_,Proj,Sg,_Sv,AbsInt,F,N,_Fs0,Id,List,Prime):-
254     current_fact(fixpoint(SgKey,Subg,Proj1,Prime1,Id,Fs),Ref),

```



```

255     identical_proj(AbsInt, Sg, Proj, Subg, Proj1), !,
256     patch_parents(Ref, fixpoint(SgKey, Subg, Proj1, Prime1, Id, Ps), F, N, Ps, Fs),
257     current_fact(ch_id(Id, Num)),
258     List = [Id/Num],
259     each_abs_sort(Prime1, AbsInt, Prime).
260 init_fixpoint1(SgKey, _Call, Proj, Sg, _Sv, AbsInt, F, N, _Fs0, Id, List, Prime):-
261     current_pp_flag(variants, on),
262     current_fact(complete(SgKey, AbsInt, Subg, Proj1, Prime1, _Id1, _Fs), _R),
263     identical_proj_1(AbsInt, Sg, Proj, Subg, Proj1, Prime1, Prime2), !,
264     ( current_pp_flag(reuse_fixp_id, on) ->
265         fixpoint_id_reuse_prev(SgKey, AbsInt, Sg, Proj, Id)
266     ;
267         fixpoint_id(Id)
268     ),
269     each_abs_sort(Prime2, AbsInt, Prime),
270     List = [],
271     asserta_fact(complete(SgKey, AbsInt, Sg, Proj, Prime, Id, [(F, N)])).
272 init_fixpoint1(SgKey, Call, Proj, Sg, Sv, AbsInt, F, N, _Fs0, Id, List, Prime):-
273     current_pp_flag(variants, on),
274     current_fact(approx(SgKey, Subg, Proj1, Prime1, Id1, Fs), Ref),
275     identical_proj_1(AbsInt, Sg, Proj, Subg, Proj1, Prime1, Prime2), !,
276     each_abs_sort(Prime2, AbsInt, TempPrime),
277     current_fact('$depend_list'(Id1, SgKey, IdList)),
278     call_to_success_approx_variant(SgKey, Subg, Call, Proj, Proj1, Sg, Sv, AbsInt, F, N, Fs,
279         Id, Id1, Ref, IdList, Prime1, TempPrime, List, Prime).
280 init_fixpoint1(SgKey, _, Proj, Sg, _Sv, AbsInt, F, N, _Fs0, Id, List, Prime):-
281     current_pp_flag(variants, on),
282     current_fact(fixpoint(SgKey, Subg, Proj1, Prime1, Id1, Fs), _Ref),
283     identical_proj_1(AbsInt, Sg, Proj, Subg, Proj1, Prime1, Prime2), !,
284     (
285         current_fact(fixpoint_variant(Id1, Id, SgKey, Sgv, Projv, Fsv), Refv),
286         identical_proj(AbsInt, Sg, Proj, Sgv, Projv) ->
287         patch_parents(Refv, fixpoint_variant(Id1, Id, SgKey, Sgv, Projv, Ps), F, N, Ps, Fsv)
288     ;
289         (
290             current_pp_flag(reuse_fixp_id, on) ->
291             fixpoint_id_reuse_prev(SgKey, AbsInt, Sg, Proj, Id)
292         ;
293             fixpoint_id(Id)
294         ),
295         asserta_fact(fixpoint_variant(Id1, Id, SgKey, Sg, Proj, [(F, N)]))
296     ),
297     each_abs_sort(Prime2, AbsInt, Prime),
298     current_fact(ch_id(Id1, Num)),
299     List = [Id1/Num].
300 init_fixpoint1(SgKey, Call, Proj, Sg, Sv, AbsInt, F, N, Fs, Id, List, Prime):-
301     init_fixpoint2(SgKey, Call, Proj, Sg, Sv, AbsInt, F, N, Fs, Id, List, Prime).
302
303 init_fixpoint2(SgKey, Call, Proj, Sg, Sv, AbsInt, F, N, Fs, Id, List, Prime):-
304     ( current_pp_flag(reuse_fixp_id, on) ->
305         fixpoint_id_reuse_prev(SgKey, AbsInt, Sg, Proj, Id)
306     ;
307         fixpoint_id(Id)
308     ),
309     asserta_fact(ch_id(Id, 1)),
310     proj_to_prime_r(SgKey, Sg, Sv, Call, Proj, AbsInt, TempPrime, Id),
311     init_fixpoint_(SgKey, Call, Proj, Sg, Sv, AbsInt, F, N, Fs, Id,
312         TempPrime, List, Prime).
313
314 init_fixpoint_(SgKey, Call, Proj, Sg, Sv, AbsInt, F, N, Fs, Id, Prime0, List, Prime):-
315     normalize_asub0(AbsInt, Prime0, TempPrime),
316     asserta_fact(fixpoint(SgKey, Sg, Proj, TempPrime, Id, Fs)),
317     bagof(X, X^(trans_clause(SgKey, r, X)), Clauses), !,
318     fixpoint_compute(Clauses, SgKey, Sg, Sv, Call, Proj,
319         AbsInt, _LEntry, TempPrime, Prime1, Id, TempList),
320     each_apply_trusted(Proj, SgKey, Sg, Sv, AbsInt, Prime1, Prime),

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```

321     current_fact(fixpoint(SgKey,Sg,_,_,Id,Fs2),Ref),
322     erase(Ref),
323     ( current_fact('$depend_list'(Id,SgKey,_,RefDep) ->
324       erase(RefDep)
325       ; true
326     ),
327     update_if_member_idlist(TempList,Id,AddList),
328     ( member((F,N),Fs2) -> NewFs = Fs2 ; NewFs = [(F,N)|Fs2] ),
329     decide_approx(AddList,Id,NewFs,AbsInt,SgKey,Sg,Proj,Prime),
330     List = AddList.
331
332 widen_call(AbsInt,SgKey,Sg,F1,Id0,Proj1,Proj):-
333     ( current_pp_flag(widencall,off) -> fail ; true ),
334     widen_call0(AbsInt,SgKey,Sg,F1,Id0,[Id0],Proj1,Proj), !.
335
336 widen_call0(AbsInt,SgKey,Sg,F1,Id0,Ids,Proj1,Proj):-
337     widen_call1(AbsInt,SgKey,Sg,F1,Id0,Ids,Proj1,Proj).
338 widen_call0(AbsInt,SgKey,Sg,F1,Id0,Ids,Proj1,Proj):-
339     current_pp_flag(widencall,com_child),
340     widen_call2(AbsInt,SgKey,Sg,F1,Id0,Ids,Proj1,Proj).
341
342 widen_call1(AbsInt,SgKey,Sg,F1,Id0,Ids,Proj1,Proj):-
343     current_fact(fixpoint(SgKey0,Sg0,Proj0,_Prime0,Id0,Fs0)),
344     ( SgKey=SgKey0,
345       % same program point:
346       member((F1,_NewId0),Fs0)
347     -> Sg0=Sg,
348       abs_sort(AbsInt,Proj0,Proj0_s),
349       abs_sort(AbsInt,Proj1,Proj1_s),
350       widencall(AbsInt,Proj0_s,Proj1_s,Proj)
351     ; % continue with the parents:
352       member((_F1,NewId0),Fs0),
353       \+ member(NewId0,Ids),
354       widen_call1(AbsInt,SgKey,Sg,F1,NewId0,[NewId0|Ids],Proj1,Proj)
355     ).
356
357 widen_call2(AbsInt,SgKey,Sg,F1,_Id,_Ids,Proj1,Proj):-
358     current_fact(complete(SgKey,AbsInt,Sg0,Proj0,_Prime0,_Id0,Fs0)),
359     member((F1,Id0),Fs0),
360     Sg0=Sg,
361     same_fixpoint_ancestor(Id0,[Id0],AbsInt),
362     abs_sort(AbsInt,Proj0,Proj0_s),
363     abs_sort(AbsInt,Proj1,Proj1_s),
364     widencall(AbsInt,Proj0_s,Proj1_s,Proj).
365
366 same_fixpoint_ancestor(Id0,_Ids,_AbsInt):-
367     current_fact(fixpoint(_SgKey0,_Sg0,_Proj0,_Prime0,Id0,_Fs0)), !.
368 same_fixpoint_ancestor(Id0,_Ids,_AbsInt):-
369     current_fact(approx(_SgKey0,_Sg0,_Proj0,_Prime0,Id0,_Fs0)), !.
370 same_fixpoint_ancestor(Id0,Ids,AbsInt):-
371     current_fact(complete(_SgKey0,AbsInt,_Sg0,_Proj0,_Prime0,Id0,Fs0)),
372     member((_F1,Id),Fs0),
373     \+ member(Id,Ids),
374     same_fixpoint_ancestor(Id,[Id|Ids],AbsInt).
375
376 fixpoint_variants_update(Id,AbsInt,Sg,Prime):-
377     current_fact(fixpoint_variant(Id,Idv,SgKey,Sgv,Projv,Fs),Ref),!,
378     erase(Ref),
379     varset(Sg,Hv),
380     varset(Sgv,Hvv),
381     each_exit_to_prime(Prime,AbsInt,Sgv,Hv,Sg,Hvv,(no,Projv),Prime2),
382     asserta_fact(complete(SgKey,AbsInt,Sgv,Projv,Prime2,Idv,Fs)),
383     fixpoint_variants_update(Id,AbsInt,Sg,Prime).
384 fixpoint_variants_update(_,_,_,_).
385
386 approx_variants_update(Id,AbsInt,Sg,Prime):-

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387     current_fact(fixpoint_variant(Id,Idv,SgKey,Sgv,Projv,Fs),Ref),!,
388     erase(Ref),
389     varset(Sg,Hv),
390     varset(Sgv,Hvv),
391     each_exit_to_prime(Prime,AbsInt,Sgv,Hv,Sg,Hvv,(no,Projv),Prime2),
392     asserta_fact(approx_variant(Id,Idv,SgKey,Sgv,Projv,Prime2,Fs)),
393     approx_variants_update(Id,AbsInt,Sg,Prime).
394 approx_variants_update(_,_,_,_).
395
396 decide_approx([],Id,Fs,AbsInt,SgKey,Sg,Proj,Prime):-!,
397     current_fact(ch_id(Id,_),Ref3),
398     erase(Ref3),
399     % Not needed for correctness: only book-keeping
400     % update_depend_list_approx(Id,AbsInt),
401     asserta_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Id,Fs)),
402     (
403         current_pp_flag(variants,on) ->
404         each_abs_sort(Prime,AbsInt,Prime_s),
405         fixpoint_variants_update(Id,AbsInt,Sg,Prime_s)
406     ;
407         true
408     ).
409 decide_approx(AddList,Id,Fs,_AbsInt,SgKey,Sg,Proj,Prime):-
410     asserta_fact('$depend_list'(Id,SgKey,AddList)),
411     asserta_fact(approx(SgKey,Sg,Proj,Prime,Id,Fs),_),
412     (
413         current_pp_flag(variants,on) ->
414         each_abs_sort(Prime,AbsInt,Prime_s),
415         approx_variants_update(Id,AbsInt,Sg,Prime_s)
416     ;
417         true
418     ).
419
420 not_modified([]).
421 not_modified([Id/N|List]):-
422     current_fact(ch_id(Id,N)),!,
423     not_modified(List).
424
425 proj_to_prime_nr(SgKey,Sg,Sv,Call,Proj,AbsInt,_ClId,LPrime,Id):-
426     bagof(X,X^(trans_clause(SgKey,nr,X)),Clauses),!,
427     proj_to_prime(Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,LPrime1,Id),
428     compute_clauses_lub(AbsInt,Proj,LPrime1,LPrime).
429 proj_to_prime_nr(SgKey,Sg,Sv,_Call,Proj,AbsInt,ClId,LPrime,_Id):-
430     apply_trusted0(Proj,SgKey,Sg,Sv,AbsInt,ClId,Prime),!,
431     singleton(Prime,LPrime).
432 proj_to_prime_nr(_SgKey,Sg,Sv,Call,_Proj,AbsInt,_ClId,LSucc,_Id):-
433     % In Java programs, mode and type information is known for any method.
434     % Therefore, in case of a method with unavailable code we can still
435     % infer useful information.
436     (current_pp_flag(prog_lang,java) ->
437         unknown_call(AbsInt,Sg,Sv,Call,Succ),
438         singleton(Succ,LSucc)
439     ;
440         fail
441     ).
442 proj_to_prime_nr(SgKey,_Sg,_Sv,_Call,_Proj,_AbsInt,ClId,Bot,_Id):-
443     bottom(Bot),
444     inexistent(SgKey,ClId).
445
446 proj_to_prime_r(SgKey,Sg,Sv,Call,Proj,AbsInt,Prime,Id):-
447     bagof(X,X^(trans_clause(SgKey,nr,X)),Clauses),!,
448     proj_to_prime(Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,Prime,Id).
449 proj_to_prime_r(_SgKey,_Sg,_Sv,_Call,_Proj,_AbsInt,Bot,_Id):-
450     bottom(Bot).
451
452 proj_to_prime(Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,Prime,Id):-

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```

453     proj_to_prime_loop(Clauses,Sg,Sv,Call,Proj,AbsInt,ListPrime0,Id),
454     reduce_equivalent(ListPrime0,AbsInt,ListPrime1),
455     each_apply_trusted(Proj,SgKey,Sg,Sv,AbsInt,ListPrime1,Prime).
456
457 proj_to_prime_loop([],_,_,_,_,[],_).
458 proj_to_prime_loop([Clause|Rest],Sg,Sv,Call,Proj,AbsInt,Primes,Id):-
459     do_nr_cl(Clause,Sg,Sv,Call,Proj,AbsInt,Primes,TailPrimes,Id),!,
460     proj_to_prime_loop(Rest,Sg,Sv,Call,Proj,AbsInt,TailPrimes,Id).
461
462 do_nr_cl(Clause,Sg,Sv,Call,Proj,AbsInt,Primes,TailPrimes,Id):-
463     Clause = clause(Head,Vars_u,K,Body),
464     clause_applies(Head,Sg), !,
465     varset(Head,Hv),
466     sort(Vars_u,Vars),
467     ord_subtract(Vars,Hv,Fv),
468     process_body(Body,K,AbsInt,Sg,Hv,Fv,Vars_u,Head,Sv,Call,
469                                                         Proj,LPrime,Id),
470     append(LPrime,TailPrimes,Primes).
471 do_nr_cl(_Clause,_Sg,_Sv,_Call,_Proj,_AbsInt,Primes,Primes,_Id).
472
473 append([Prime],TailPrimes,Primes):- !, Primes=[Prime|TailPrimes].
474 append(LPrime,TailPrimes,Primes):- append(LPrime,TailPrimes,Primes).
475
476 process_body(Body,K,AbsInt,Sg,Hv,Fv,_,Head,Sv,Call,Proj,LPrime,Id):-
477     Body = g(_,[],'$built'(_,true,_),'true/0',true), !,
478     Help=(Sv,Sg,Hv,Fv,AbsInt),
479     singleton(Prime,LPrime),
480     call_to_success_fact(AbsInt,Sg,Hv,Head,K,Sv,Call,Proj,Prime,_Succ),
481     ( current_pp_flag(fact_info,on) ->
482       call_to_entry(AbsInt,Sv,Sg,Hv,Head,K,[],Prime,Exit,_),
483       decide_memo(AbsInt,K,Id,no,Hv,[Exit])
484     );
485     true
486   ).
487 process_body(Body,K,AbsInt,Sg,Hv,Fv,Vars_u,Head,Sv,_,Proj,Prime,Id):-
488     call_to_entry(AbsInt,Sv,Sg,Hv,Head,K,Fv,Proj,Entry,ExtraInfo),
489     singleton(Entry,LEntry),
490     entry_to_exit(Body,K,LEntry,Exit,[],_,Vars_u,AbsInt,Id),
491     each_exit_to_prime(Exit,AbsInt,Sg,Hv,Head,Sv,ExtraInfo,Prime).
492
493 fixpoint_compute(Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,
494                 Prime0,Prime,Id,List) :-
495     fixpoint_compute_(Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,
496                     Prime0,Prime1,Id,List),
497     compute_clauses_lub(AbsInt,Proj,Prime1,Prime).
498
499 fixpoint_compute_(Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,
500                  TempPrime,Prime,Id,List) :-
501     compute(Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,
502            TempPrime,Prime1,Id,[],NewList,Flag),
503     fixpoint(NewList,Flag,Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,
504            Prime1,Prime,Id,List), !.
505
506 fixpoint([],_,_,_,_,_,_,_,_,Prime1,Prime,_,List):- !,
507     Prime = Prime1,
508     List = [].
509 fixpoint(NewList,Flag,_,_,_,_,_,_,_,Prime1,Prime,_,List):-
510     var(Flag),!,
511     Prime = Prime1,
512     List = NewList.
513 fixpoint(_,_,Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,Prime1,Prime,Id,List):-
514     fixpoint_compute_(Clauses,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,
515                     Prime1,Prime,Id,List).
516
517 % some domains need normalization to perform the widening:
518 normalize_asub0(AbsInt,Prime0,Prime):-

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```

519     current_pp_flag(widen,on), !,
520     normalize_asub(AbsInt,Prime0,Prime).
521 normalize_asub0(_AbsInt,Prime,Prime).
522
523 compute([],_,_,_,_,_,[],Prime,Prime,_,List,List,_).
524 compute([Clause|Rest],SgKey,Sg,Sv,Call,Proj,AbsInt,[EntryInf|LEntryInf],
525         TempPrime,Prime,Id,List,NewList,Flag) :-
526     do_r_cl(Clause,SgKey,Sg,Sv,Proj,AbsInt,EntryInf,Id,List,IntList,
527            TempPrime,NewPrime,Flag),
528     compute(Rest,SgKey,Sg,Sv,Call,Proj,AbsInt,LEntryInf,NewPrime,Prime,
529            Id,IntList,NewList,Flag).
530
531 do_r_cl(Clause,SgKey,Sg,Sv,Proj,AbsInt,EntryInf,Id,OldL,List,TempPrime,
532         NewPrime,Flag):-
533     Clause=clause(Head,Vars_u,K,Body),
534     clause_applies(Head,Sg), !,
535     erase_previous_memo_tables_and_parents(Body,AbsInt,K,Id),
536     varset(Head,Hv),
537     reuse_entry(EntryInf,Vars_u,AbsInt,Sv,Sg,Hv,Head,K,Proj,Entry,ExtraInfo),
538     singleton(Entry,LEntry),
539     entry_to_exit(Body,K,LEntry,Exit,OldL,List,Vars_u,AbsInt,Id),
540     each_exit_to_prime(Exit,AbsInt,Sg,Hv,Head,Sv,ExtraInfo,Prime1),
541     widen_succ(AbsInt,TempPrime,Prime1,NewPrime),
542     decide_flag(AbsInt,TempPrime,NewPrime,SgKey,Sg,Id,Proj,Flag).
543
544 do_r_cl(_,_,_,_,_,_,_,List,List,Prime,Prime,_).
545
546 widen_succ_off(AbsInt,Prime0,Prime1,LPrime):-
547     current_pp_flag(multi_success,on), !,
548     reduce_equivalent([Prime0,Prime1],AbsInt,LPrime).
549 widen_succ_off(AbsInt,Prime0,Prime1,Prime):-
550     singleton(P0,Prime0),
551     singleton(P1,Prime1),
552     singleton(P,Prime),
553     compute_lub(AbsInt,[P0,P1],P).
554
555 reuse_entry(EntryInf,Vars_u,AbsInt,Sv,Sg,Hv,Head,K,Proj,Entry,ExtraInfo):-
556     var(EntryInf), !,
557     sort(Vars_u,Vars),
558     ord_subtract(Vars,Hv,Fv),
559     call_to_entry(AbsInt,Sv,Sg,Hv,Head,K,Fv,Proj,Entry,ExtraInfo),
560     EntryInf = (Entry,ExtraInfo).
561 reuse_entry(EntryInf,_Vars_u,_AbsInt,_Sv,_Sg,_Hv,_Head,_K,_Proj,Entry,ExtraInfo):-
562     EntryInf = (Entry,ExtraInfo).
563
564 decide_flag(AbsInt,TempPrime,NewPrime,_SgKey,_Sg,_Id,_Proj,_Flag):-
565     abs_subset_(NewPrime,AbsInt,TempPrime), !.
566 decide_flag(_AbsInt,TempPrime,NewPrime,SgKey,Sg,Id,Proj,Flag):-
567     Flag = notend,
568     merge_(NewPrime,TempPrime,LPrime),
569     current_fact(fixpoint(SgKey,Sg,_,_,Id,Fs),Ref),
570     erase(Ref),
571     asserta_fact(fixpoint(SgKey,Sg,Proj,LPrime,Id,Fs)),
572     current_fact(ch_id(Id,Num),Ref3),
573     erase(Ref3),
574     Num1 is Num+1,
575     asserta_fact(ch_id(Id,Num1)).
576
577 merge_([NewPrime],_TempPrime,LPrime):- !, LPrime=[NewPrime].
578 merge_(NewPrime,TempPrime,LPrime):-
579     merge(NewPrime,TempPrime,LPrime).
580
581 %-----%
582 % entry_to_exit(+,+,+,-,+,-,+,+,+) %
583 %-----%
584

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```

585 entry_to_exit((Sg,Rest),K,Call,Exit,OldList,NewList,Vars_u,AbsInt,NewN):-!,
586   body_succ(Call,Sg,Succ,OldList,IntList,Vars_u,AbsInt,K,NewN,_),
587   entry_to_exit(Rest,K,Succ,Exit,IntList,NewList,Vars_u,AbsInt,NewN).
588 entry_to_exit(true,_,Call,Call,List,List,_,_,_):-!.
589 entry_to_exit(Sg,Key,Call,Exit,OldList,NewList,Vars_u,AbsInt,NewN):-
590   body_succ(Call,Sg,Exit,OldList,NewList,Vars_u,AbsInt,Key,NewN,_),
591   decide_memo(AbsInt,Key,NewN,no,Vars_u,Exit),!.
592
593 body_succ(Call,Atom,Succ,List,List,HvFv_u,AbsInt,_ClId,ParentId,no):-
594   bottom(Call),!,
595   Succ = Call,
596   Atom=g(Key,_Av,_I,_SgKey,_Sg),
597   asserta_fact(memo_table(Key,AbsInt,ParentId,no,HvFv_u,Succ)).
598 body_succ(Call,Atom,Succ,List,NewList,HvFv_u,AbsInt,ClId,ParentId,Id):-
599   Atom=g(Key,Sv,Info,SgKey,Sg),
600   body_succ_(Info,SgKey,Sg,Sv,HvFv_u,Call,Succ,List,NewList,AbsInt,
601     ClId,Key,ParentId,Id),
602   decide_memo(AbsInt,Key,ParentId,Id,HvFv_u,Call).
603
604 body_succ_(Info,SgKey,Sg,Sv,HFv,Call,Succ,L,NewL,AbsInt,ClId,Key,PIId,Id):-
605   Info = [_|_],!,
606   split_combined_domain(AbsInt,Call,Calls,Domains),
607   map_body_succ(Info,SgKey,Sg,Sv,HFv,Calls,Succs,L,NewL,Domains,
608     ClId,Key,PIId,Id),
609   split_combined_domain(AbsInt,Succ,Succs,Domains).
610 body_succ_(Info,SgKey,Sg,Sv,HFv,Call,Succ,L,NewL,AbsInt,ClId,Key,PIId,Id):-
611   body_succ0(Info,SgKey,Sg,Sv,HFv,Call,Succ,L,NewL,AbsInt,
612     ClId,Key,PIId,Id).
613
614 map_body_succ([],_SgKey,_Sg,_Sv,_HFv,[],[],L,L,[],_ClId,_Key,_PIId,no).
615 map_body_succ([I|Info],SgKey,Sg,Sv,HFv,[Call|Calls],[Succ|Succs],L,NewL,
616   [AbsInt|Domains],ClId,Key,PIId,Id):-
617   body_succ0(I,SgKey,Sg,Sv,HFv,Call,Succ,L,_NewL,AbsInt,
618     ClId,Key,PIId,_Id),!,
619   map_body_succ(Info,SgKey,Sg,Sv,HFv,Calls,Succs,L,NewL,Domains,
620     ClId,Key,PIId,Id).
621
622 body_succ0('$var',SgKey,Sg,_Sv_u,HvFv_u,Calls,Succs,List0,List,AbsInt,
623   ClId,F,_N,Id):-
624   !,
625   (Calls=[Call],
626     concrete(AbsInt,Sg,Call,Concretes),
627     concretes_to_body(Concretes,SgKey,AbsInt,B)
628   -> fixpoint_id(Id),
629     meta_call(B,HvFv_u,Calls,[],Succs,List0,List,AbsInt,ClId,Id,Ids),
630     assertz_fact(memo_call(F,Id,AbsInt,Concretes,Ids))
631   ; Id=no,
632     List=List0,
633     variable(F,ClId),
634     each_unknown_call(Calls,AbsInt,[Sg],Succs) % Sg is a variable
635   ).
636 body_succ0('$meta'(T,B,_),SgKey,Sg,Sv_u,HvFv_u,Call,Succ,List0,List,AbsInt,
637   ClId,F,N,Id):-
638   !,
639   (current_pp_flag(reuse_fixp_id,on) ->
640     (Call=[C]
641       -> sort(Sv_u,Sv),
642         project(AbsInt,Sg,Sv,HvFv_u,C,Proj),
643         fixpoint_id_reuse_prev(SgKey,AbsInt,Sg,Proj,Id)
644       ; true
645     )
646   ;
647     fixpoint_id(Id)
648   ),
649   meta_call(B,HvFv_u,Call,[],Exits,List0,List,AbsInt,ClId,Id,_Ids),
650   (body_succ_meta(T,AbsInt,Sv_u,HvFv_u,Call,Exits,Succ) ->

```

```

651     ( Call=[C] ->
652       sort(Sv_u,Sv),
653       project(AbsInt,Sg,Sv,HvFv_u,C,Proj),
654       each_project(Exits,AbsInt,Sg,Sv,HvFv_u,Prime),
655       asserta_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Id,[(F,N)]))
656     ; true
657   )
658 ; % for the trusts, if any:
659 varset(Sg,Sv_r), % Sv_u contains extra vars (from meta-term)
660                % which will confuse apply_trusted
661 body_succ0(nr,SgKey,Sg,Sv_r,HvFv_u,Call,Succ,[],_List,AbsInt,
662           ClId,F,N,Id0),
663 retract_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Id0,Ps)),
664 asserta_fact(complete(SgKey,AbsInt,Sg,Proj,Prime,Id,Ps))
665 ).
666 body_succ0('$built'(T,Tg,Vs),SgKey,Sg,Sv_u,HvFv_u,Call,Succ,List0,List,AbsInt,
667           _ClId,F,N,Id):-
668   !,
669   Id=no,
670   List=List0,
671   sort(Sv_u,Sv),
672   each_body_succ_builtin(Call,AbsInt,T,Tg,Vs,SgKey,Sg,Sv,HvFv_u,F,N,Succ).
673 body_succ0(RFlag,SgKey,Sg,Sv_u,HvFv_u,Call,Succ,List0,List,AbsInt,
674           ClId,F,N,Id):-
675   sort(Sv_u,Sv),
676   each_call_to_success(Call,RFlag,SgKey,Sg,Sv,HvFv_u,AbsInt,ClId,
677                       Succ,List0,List,F,N,Id).
678
679 each_call_to_success([Call],RFlag,SgKey,Sg,Sv,HvFv_u,AbsInt,ClId,Succ,L0,L,
680                     F,N,Id):-
681   !,
682   project(AbsInt,Sg,Sv,HvFv_u,Call,Proj),
683   call_to_success(RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,ClId,Succ,L1,F,N,Id),
684
685   merge(L1,L0,L).
686 each_call_to_success(LCall,RFlag,SgKey,Sg,Sv,HvFv_u,AbsInt,ClId,LSucc,L0,L,
687                     F,N,Id):-
688   each_call_to_success0(LCall,RFlag,SgKey,Sg,Sv,HvFv_u,AbsInt,ClId,
689                        LSucc,L0,L,F,N,Id).
690
691 each_call_to_success0([],_Flag,_SgK,_Sg,_Sv,_HvFv,_AbsInt,_,[],L,L,_F,_N,_NN).
692 each_call_to_success0([Call|LCall],RFlag,SgKey,Sg,Sv,HvFv_u,AbsInt,ClId,
693                      LSucc,L0,L,F,N,NewN):-
694   project(AbsInt,Sg,Sv,HvFv_u,Call,Proj),
695   call_to_success(RFlag,SgKey,Call,Proj,Sg,Sv,AbsInt,ClId,LSucc0,L1,F,N,_),
696   merge(L0,L1,L2),
697   append(LSucc0,LSucc1,LSucc),
698   each_call_to_success0(LCall,RFlag,SgKey,Sg,Sv,HvFv_u,AbsInt,ClId,
699                        LSucc1,L2,L,F,N,NewN).
700
701 meta_call([],_HvFv_u,Call,[],Call,List,List,_AbsInt,_ClId,_Id,[]).
702 meta_call([Body|Bodies],HvFv_u,Call,Succ0,Succ,L0,List,AbsInt,ClId,Id,Ids):-
703   meta_call_([Body|Bodies],HvFv_u,Call,Succ0,Succ,L0,List,AbsInt,ClId,Id,Ids).
704
705 meta_call_([Body|Bodies],HvFv_u,Call,Succ0,Succ,L0,List,AbsInt,ClId,Id,Ids):-
706   meta_call_body(Body,ClId,Call,Succ1,L0,L1,HvFv_u,AbsInt,Id,Ids0),
707   widen_succ(AbsInt,Succ0,Succ1,Succ2),
708   append(Succ0,Succ1,Succ2),
709   append(Ids0,Ids1,Ids),
710   meta_call_(Bodies,HvFv_u,Call,Succ2,Succ,L1,List,AbsInt,ClId,Id,Ids1).
711 meta_call_([],_HvFv_u,_Call,Succ,Succ,List,List,_AbsInt,_ClId,_Id,[]).
712
713 meta_call_body((Sg,Rest),K,Call,Exit,OldList,NewList,Vars_u,AbsInt,PId,CIDs):-
714   !,
715   CIDs=[Id|Ids],
716   body_succ(Call,Sg,Succ,OldList,IntList,Vars_u,AbsInt,K,PId,Id),

```

```

717     meta_call_body(Rest,K,Succ,Exit,IntList,NewList,Vars_u,AbsInt,PId,Ids).
718 meta_call_body(true,_,Call,Call,List,List,_,_,_,[no]):-!.
719 meta_call_body(Sg,Key,Call,Exit,OldList,NewList,Vars_u,AbsInt,PId,[Id]):-
720     body_succ(Call,Sg,Exit,OldList,NewList,Vars_u,AbsInt,Key,PId,Id).
721
722 concretes_to_body([],_SgKey,_AbsInt,[]).
723 concretes_to_body([Sg|Sgs],SgKey,AbsInt,[B|Bs]):-
724     body_info0(Sg:SgKey,[],AbsInt,B),
725     concretes_to_body(Sgs,SgKey,AbsInt,Bs).
726
727 %------%
728 % query(+,+,+,+,+,+,+,+) %
729 %------%
730
731 :- doc(query(AbsInt,QKey,Query,Qv,RFlag,N,Call,Succ),
732     "The success pattern of @var{Query} with @var{Call} is
733     @var{Succ} in the analysis domain @var{AbsInt}. The predicate
734     called is identified by @var{QKey}, and @var{RFlag} says if it
735     is recursive or not. The goal @var{Query} has variables @var{Qv},
736     and the call pattern is uniquely identified by @var{N}.").
737
738 query(AbsInt,QKey,Query,Qv,RFlag,N,Call,Succ) :-
739     project(AbsInt,Query,Qv,Qv,Call,Proj),
740     call_to_success(RFlag,QKey,Call,Proj,Query,Qv,AbsInt,0,Succ,_,N,0,Id),!,
741     approx_to_completes(AbsInt).
742
743 query(_AbsInt,_QKey,_Query,_Qv,_RFlag,_N,_Call,_Succ):-
744     % should never happen, but...
745     error_message("SOMETHING HAS FAILED!").

```