

Parallel Backtracking with Answer Memoing for Independent And-Parallelism

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Introduction

- Two main sources of parallelism in LP:
 - ▶ OR-Parallelism: Aurora, MUSE, etc.
 - ▶ AND-Parallelism: &-Prolog, DDAS, etc.
 - ▶ Both: (&)ACE, AKL, Andorra-I, EAM, etc.
- Classical approach in Independent AND-Parallelism (IAP):
 - ▶ Conery approach:
 - ★ Copying goals overhead.
 - ★ Nonterminating programs.
 - ▶ Recomputation + sequential backtracking.
Saves memory and preserves sequential semantics, but...
 - ★ Recomputation can be inefficient.
 - ★ Sequential backtracking limits parallelism.

Classical approach in IAP

Example (Traditional IAP execution - main1 vs. main2)

```
main1(X, Y) :-  
    g1(X), g2(Y).
```

```
g1(X) :- X=1, work(2).  
g1(X) :- X=2, work(10).
```

```
main2(X, Y) :-  
    g1(X) & g2(Y).
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```
g2(Y) :- Y=1, work(2).  
g2(Y) :- Y=2, work(5).  
g2(Y) :- Y=3, work(5).
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- $\text{Time}_{\text{main1}} = \overbrace{2}^{g1}$

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- $\text{Time}_{\text{main1}} = \overbrace{2}^{\text{g1}} + \overbrace{(2 + 5)}^{\text{g2}}$

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- $\text{Time}_{\text{main2}} = \overbrace{\max(2,2)}^{g1\&g2} + \overbrace{(5+5)}^{g2} + \overbrace{\max(10,2)}^{g1\&g2} + \overbrace{(5+5)}^{g2} = 32 \text{ secs.}$
- Speedup = 1.12. Can we do better?

Our approach for IAP

- We propose an improved solution to backtracking in IAP, which combines:
 - ▶ Parallel backtracking.
 - ▶ Answer memoing.
 - ▶ Incremental computation of answers.
- We maintain high-level approach [ICLP'08].
 - ▶ Easier to maintain and extend.

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- Time_{main1} = 2 + (2 + 5 + 5) + 10 + (2 + 5 + 5) = 36 secs.

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- Time_{main1} = 2 + (2 + 5 + 5) + 10 + (2 + 5 + 5) = 36 secs.
- Time_{main1} with memoization = $\overbrace{2}^{g1}$

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- $\text{Time}_{\text{main1}} = 2 + (2 + 5 + 5) + 10 + (2 + 5 + 5) = 36 \text{ secs.}$
- $\text{Time}_{\text{main1}} \text{ with memoization} = \overbrace{2}^{g1} + \overbrace{(2 + 5 + 5)}^{g2}$

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- $\text{Time}_{\text{main1}} = 2 + (2 + 5 + 5) + 10 + (2 + 5 + 5) = 36 \text{ secs.}$
- $\text{Time}_{\text{main1}} \text{ with memoization} = \overbrace{2}^{g1} + \overbrace{(2 + 5 + 5)}^{g2} + \overbrace{10}^{g1} = 24 \text{ secs.}$

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- Time_{main1} = 2 + (2 + 5 + 5) + 10 + (2 + 5 + 5) = 36 secs.
- Time_{main1} with memoization = $\overbrace{2}^{g1} + \overbrace{(2 + 5 + 5)}^{g1\&g2} + \overbrace{10}^{g1} = 24$ secs.
- Time_{main2} with memoization = $\overbrace{\max(2, 2)}^{g1\&g2} + \overbrace{\max(10, 5 + 5)}^{g1\&g2} = 12$ secs.
- Speedup w.r.t. sequential with memoization = 2.00.
- Speedup w.r.t. sequential = 3.00 (superlinear).

Our approach for IAP - Forward execution

- *Forward execution:* similar to classical IAP.
 - ▶ If a goal fails for first solution, conjunction fails (no slow-down!).
 - ▶ If/when all goals find a solution, execution proceeds.
- *Speculation:* if conjunction not finished yet, additional solutions are sought for goals that already computed an answer (if free agents).
 - ▶ Need to stash away generated solutions to continue searching for more answers (which are also saved).
- *Suspension:* when all goals find a solution, any speculative goals are suspended, their state saved, and their first answer reinstalled.
 - ▶ New **answers saved in memoing area** to avoid future recomputation.
 - ▶ Every new solution is **combined with** previously available solutions.

Our approach for IAP - Backward execution

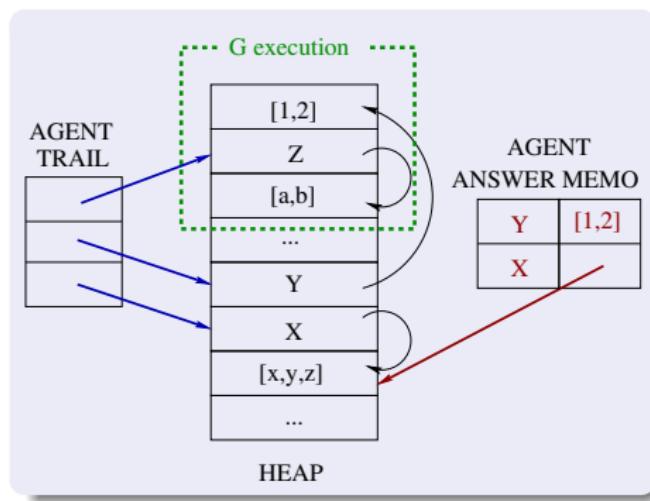
- *Parallel backtracking*: if more solutions needed, backward execution performed in parallel. Suspended goals resume.
- *Backward execution*: performed over goals on top of the stack.
 - ▶ More efficient implementation, i.e., *trapped goals very infrequent*.
 - ▶ Solution order differs from sequential execution.

Memoization of Answers

- Goals are independent:
 - ▶ Makes sense to store answers to avoid recomputation (we know better how to do this now).
 - Answer memoing different/simpler from tabling:
 - ▶ Termination not an issue. No need for detecting repeated calls, suspending/resuming consumers, etc.
 - ▶ All stored answers are discarded as soon as the conjunction continues after its last answer, or after the conjunction fails.
 - ▶ Visibility of stored answers restricted to only the parallel conjunction.
- Easier case to implement/maintain than tabling.

Memoization of Answers - Implementation

- Answer memoization = saving a combination of heap and trail terms.
 - ▶ Trail segment corresponding to execution of parallel goal, and terms created during the goal execution pointed to by these variables.



- ▶ Reinstalling an answer consists of copying saved terms back to heap, and updating the trail.
- ▶ Memoization has a cost, but it can provide by itself substantial speedups since it avoids recomputation.

Answer Combination

- When last goal pending to generate an answer in a conjunction produces a solution, it needs to be combined (from right to left) with the rest of the answers to continue with forward execution.

Example (Combining answers in a parallel conjunction)

```
main(X, Y, Z) :-  
    g1(X) & g2(Y) & g3(Z).
```

g1(a). g2(x).	g1(b). g2(y).	g1(c). g2(z).
g3(1). g3(2).		g3(3).

- Assume $g1$ and $g2$ have computed two answers, and $g3$ only one.
- Then, $g3$ finds a new answer $\{Z = 2\}$.
 - First combination of answers will be $(a, x, 2)$.
 - Next combination will be $(a, y, 2)$.
 - Final combinations will be $(b, x, 2)$ and $(b, y, 2)$.

Answer Combination - Implementation

- A *ghost* choice point is created with an alternative that retrieves the saved answers and installs the bindings.
 - ▶ Heap top pointer of ghost choice point points to the current heap top after copying terms, to protect those terms from backtracking for future answer combinations.
 - ▶ All these copied terms are released when the ghost choice point is eliminated.

Scheduler for the Parallel Backtracking IAP Engine

```

agent :- work, agent.

work :-
    find_parallel_goal(Handler) ->
    (
        goal_not_executed(Handler) ->
        (
            save_init_execution(Handler),
            call_parallel_goal(Handler)
        ;
            move_execution_top(Handler),
            fail
        )
    ;
        suspend,
        work
    ).

```

```

parcall_back(LGoals, NGoals) :-  

    fork(PF,NGoals,LGoals,[Handler|LH]),
    (  

        goal_not_executed(Handler) ->  

        call_local_goal(Handler,Goal)  

    ;  

        true  

    ),  

    look_for_available_goal(LH),  

    join(PF).  
  

look_for_available_goal([]) :- !, true.  

look_for_available_goal([Handler|LH]) :-  

    (  

        goal_available(Handler) ->  

        call_local_goal(Handler,Goal)  

    ;  

        true  

    ),  

    look_for_available_goal(LH).

```

Deterministic Parallel Goals

- Machinery proposed can be greatly simplified if goals deterministic.
 - ▶ Answer memoization and answer combination are not needed.
 - ▶ High-level scheduler can be greatly simplified.
- Some optimizations can be performed dynamically.
- Performance improvements of up to a factor of two in the execution of deterministic benchmarks.

Deterministic Benchmarks - Sun Fire T2000

- 8 cores x 4 threads, 8 Gb of memory, average of 10 runs.
- Speedups w.r.t. sequential (unparallelized) execution.

Benchmark	Approach	Number of processors				
		1	2	4	6	8
Fibo	<i>&-Prolog</i>	0.98	1.93	3.70	5.65	7.34
	<i>seqback</i>	0.95	1.89	3.70	5.36	6.96
	<i>parback</i>	0.95	1.88	3.69	5.33	6.94
	<i>parback_{det}</i>	0.96	1.91	3.74	5.41	7.04
MMatrix	<i>&-Prolog</i>	1.00	1.99	3.98	5.96	7.93
	<i>seqback</i>	0.78	1.55	2.99	4.29	5.55
	<i>parback</i>	0.76	1.52	2.95	4.22	5.45
	<i>parback_{det}</i>	0.80	1.60	3.01	4.55	5.87
QSort	<i>&-Prolog</i>	1.00	1.92	3.03	3.89	4.65
	<i>seqback</i>	0.50	0.98	1.74	2.27	2.67
	<i>parback</i>	0.49	0.97	1.74	2.27	2.69
	<i>parback_{det}</i>	0.56	1.10	1.96	2.57	3.02
	<i>seqbackGC</i>	0.97	1.77	3.02	3.77	4.15
	<i>parbackGC</i>	0.97	1.76	3.00	3.74	4.12
	<i>parbackGC_{det}</i>	0.97	1.78	3.04	3.79	4.21

Nondeterministic Benchmarks - Sun Fire T2000

Benchmark	Approach	Number of processors				
		1	2	4	6	8
CheckFiles	$seqback_{first}$	0.99	1.09	1.12	1.12	1.13
	$seqback_{all}$	0.99	1.05	1.07	1.08	1.08
	$parback_{first}$	3917	8612	17111	17116	44222
	pb_rel_{first}	1.00	2.20	4.37	4.37	11.29
	$parback_{all}$	12915	23409	45818	46955	89571
	pb_rel_{all}	1.00	1.81	3.55	3.64	6.94
Illumination	$seqback_{first}$	1.00	1.37	1.56	1.61	1.67
	$seqback_{all}$	1.00	1.16	1.24	1.25	1.27
	$parback_{first}$	1120	1725	3380	4028	6910
	pb_rel_{first}	1.00	1.54	3.02	3.60	6.17
	$parback_{all}$	8760	16420	31818	31888	65314
	pb_rel_{all}	1.00	1.87	3.63	3.64	7.46
QSortND	$seqback_{first}$	0.94	1.72	2.92	3.59	3.92
	$seqback_{all}$	0.91	0.96	0.99	1.00	1.00
	$parback_{first}$	0.94	1.72	2.91	3.57	3.91
	$parback_{all}$	4.29	6.27	9.90	10.90	11.30
	pb_rel_{all}	1.00	1.46	2.31	2.54	2.64

Conclusions

- Developed a parallel (out-of-order) backtracking approach for IAP, which relies on answer memoization to reuse and combine answers.
- Our approach provides significant performance improvements in the execution of nondeterministic parallel calls, due to the answer memoization mechanism and parallel backtracking.
- Optimizations allow avoiding the overhead for deterministic goals.