Issues in Parallel Execution of Non-monotononic Reasoning Systems

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1 Introduction

In recent years we have witnessed a rapid development of logical systems—*non-monotonic logics*—that provide the ability to retract existing theorems via introduction of new axioms. In the context of logic programming, non-monotonic behavior has been accomplished by allowing the use of negation in the body of clauses. The presence of negation leads to a natural support for non-monotonic reasoning, allowing for intelligent reasoning in presence of incomplete knowledge. Negation is also important for various forms of database technology (e.g., deductive databases).

Stable model semantics [20] is one of the most commonly accepted approaches to provide semantics to logic programs with negation. Stable model semantics relies on the idea of accepting multiple minimal models as a description of the meaning of a program. In spite of its wide acceptance and its extensive mathematical foundations, stable models semantics have only recently found its way into mainstream "practical" logic programming. The recent successes have been sparked by the availability of very efficient inference engines (such as *smodels* [34], DeRes [9], and DLV [17]) and a substantial effort towards *understanding* how to write programs under stable models semantics [33, 30, 26]. This has lead to the development of a novel *programming paradigm*, commonly referred to as *Answer Set Programming (ASP)*. ASP is a computation paradigm in which logical theories (Horn clauses with negation) serve as problem specifications and solutions are represented by *collection of models*. ASP has been concretized in a number of related formalism—e.g., disjunctive logic programming and Datalog with constraints [17, 14]. In comparison to other non-monotonic logics, ASP is syntactically simpler and, at the same time, very expressive. The mathematical foundations of ASP have been extensively studied; in addition, there exist a large number of *building block* results about specifying and programming using ASP—e.g., results about dealing with incomplete information and abductive assimilation of new knowledge. ASP has been successfully adopted in various domains (e.g., [26, 24, 36]).

In spite of the continuous effort in developing fast execution models for ASP [17, 14, 34], computation of significant programs remains a challenging task, limiting the scope of applicability of ASP in a number of domains (e.g., planning). In this work we propose the use of *parallelism* to improve performance of ASP engines and improve the scope of applicability of this paradigm. The core of our work is the identification of a number of potential sources for *implicit* exploitation of parallelism from a basic execution model for ASP programs—specifically the execution model proposed in the *smodels* system [34]. We show that ASP has the potential to provide considerable amounts of independent tasks, which can be concurrently explored by different ASP engines. Exploitation of parallelism can be accomplished in a fashion similar to the models proposed to parallelize Prolog [23] and constraint propagation [32].

In this paper we overview the main issues in the exploitation of parallelism from the basic execution model of ASP. We identify two major forms of parallelism, *Horizontal* parallelism and *Vertical* parallelism, that respectively correspond to the two instances of non-determinism present in the propagation-based operational semantics commonly used for ASP. Building on recent theoretical results regarding the efficiency of parallel development of search trees [40, 38], we investigate the development of techniques to handle the different forms of parallelism in an ASP engine and we present preliminary experimental results accomplished.

The work proposed—along with the work concurrently conducted by Finkel et al. [19]—represents the first exploration in the use of scalable architectures for ASP computations ever proposed.

2 Non-monotonic Reasoning and Answer Set Programming

2.1 Answer Set Semantics

Answer Sets Semantics (AS) [20] (a.k.a. *Stable Models* semantics) was designed in the mid 80s as a tool to provide semantics for LP with *negation* [4]. Traditional LP [29] provides the ability to derive only positive consequences from a program. However, in a large number of cases it is useful to reason also about negative consequences, by allowing negative knowledge to be inferred and allowing negative assumptions in the rules of the program. The presence of negation leads to a natural support for non-monotonic reasoning, and the availability of efficient computational mechanisms provides a natural setting for the study of proof systems for non-monotonic reasoning.

The introduction of negation in logic programming leads to a number of complications, starting from the fact that negation may lead to the loss of one of the key properties of standard LP, the existence of a unique intended model for each program—i.e., intuitively, there is no ambiguity in what is true and what is false w.r.t. the program. Two classes of proposals have been developed to tackle the problem of providing semantics to logic programs in presence of negation.

The first class [3, 47] attempts to reconstruct a single intended model, either by narrowing the class of admissible programs (e.g., *stratified programs* [44]) or by switching to 3-valued semantics—i.e., admitting the fact that formulae can be not only true or false, but also undefined. The second direction of research instead admits the existence of a *collection* of intended models for a program [10, 20]. Answer sets is the most representative approach in this second class. AS has been recognized to provide the right semantics for LP with negation—e.g., it subsumes the intended model of logic programs without negation, it subsumes the intended model in the approach based on stratification [4], etc.

AS relies on a very simple definition. Given a program P[3] and given a tentative model M, we can define a new program P^M (the *reduct* of P w.r.t. M) which is obtained by

- removing all rules containing negative elements which are contradicted by the model M;
- removing all negative elements from the remaining rules.

Thus, P^M contains only those rules of P that are applicable given the model M. Furthermore, P^M is a standard logic program, without negation, which admits a unique intended model M' [29]. M is an *answer set* (a.k.a. *stable model*) if M and M' coincide. Intuitively, a stable model contains all and only those atoms which have a justification in terms of the applicable rules in the program. These models can be proved to be *minimal*, and in general a program with negation may admit more than one answer set.

Example 1. If we have a database indicating people working in different departments

```
dept(hartley,cs).        dept(pfeiffer,cs).
dept(gerke,math).        dept(prasad,ee).
```

and we would like to select the existing departments and one representative employee for each of them:

```
depts_employee(Name,Dep) :- dept(Name,Dep), not other_emps(Name,Dep).
other_emps(Name,Dep)     :- dept(Name1,Dep), depts_employees(Name1,Dep),
                            Name ≠ Name1.
```

The rules assert that `Name/Dep` should be added to the solution only if no other member of the same department has been selected. AS produces for this program 2 answer sets:

$$\{\langle \texttt{hartley}, \texttt{cs}\rangle, \langle \texttt{gerke}, \texttt{math}\rangle, \langle \texttt{prasad}, \texttt{ee}\rangle\}$$
$$\{\langle \texttt{pfeiffer}, \texttt{cs}\rangle, \langle \texttt{gerke}, \texttt{math}\rangle, \langle \texttt{prasad}, \texttt{ee}\rangle\}$$

[3] Let us assume for the sake of simplicity that it does not contain variables.

2.2 Answer Sets Programming: a Novel Paradigm

As recognized by a number of authors [30, 33], the adoption of AS requires a *paradigm shift* to reconcile the peculiar features of AS with the traditional program view of logic programming. This need arises for a number of reasons. In first place, under AS, each program potentially admits more than one intended model. This ends up creating an additional level of non-determinism—specifically a form of *don't know* non-determinism—on top of the forms of non-determinism typically identified in traditional LP (and directly ensuing from the use of resolution). The presence of multiple answer sets complicates the framework in two ways. First of all, we need to provide programmers with a way of handling the multiple answer sets. On one hand, one could attempt to restore a more "traditional" view, where a single "model" exists. This has been attempted, for example, using *skeptical semantics* [30], where an atom is considered entailed from the program only if it is true in each answer set. For certain classes of programs skeptical semantics coincides with other semantics proposed for LP with negation. Nevertheless, skeptical semantics is often inadequate—e.g., in many situations it does not provide the desired result (see example 1), and in its general form provides excessive expressive power [30, 31]. The additional level of non-determinism—removed by skeptical semantics—is indeed a real need for a number of applications; it is also possible to see some similarities between this and some of the proposals put forward in other communities—such as the *choice* and *witness* constructs used in the database community [25, 1, 41].

The presence of multiple answer sets, in turn, leads to a new set of requirements on the *computational mechanisms* used. Given a program, now the main goal of the computation is not to provide a goal-directed tuple-at-a-time answer (i.e., a true/false answer or an answer substitution), as in traditional LP, but the goal is to return *whole answer sets*. The traditional resolution-based control used in LP is largely inadequate, and should give place to a different form of control and different execution mechanisms.

In this project we embrace a different view of LP under AS, interpreted as a *novel programming paradigm*—that we will refer to as *Answer Sets Programming (ASP)*. This term was originally created by V. Lifschitz, and nicely blends the notion of programming with the idea that the entities produced by the computation are answer sets. The notion of ASP is not completely new and has been advocated by others during the last two years: Niemela has recently proposed answer sets semantics as a *constraint programming paradigm* [33], while Marek & Truszczynski have coined the term *Stable Logic Programming* [30] to capture the notion we are describing.

In simple terms, the goal of an ASP program is to identify a *collection of answer sets*—i.e., each program is interpreted as a specification of a collection of *sets of atoms*. Each rule in the program plays the role of a *constraint* [33] on the collection of sets specified by the program: a generic rule

$$Head :- B_1, \ldots, B_n, not\ G_1, \ldots, not\ G_m$$

requires that whenever $B_1, \ldots, B_n$ are part of the answer set and $G_1, \ldots, G_m$ are not, then *Head* has to be in the answer set as well. The shift of perspective from LP to ASP is very important. The programmer is lead to think about writing programs as manipulating sets of elements, and the outcome of the computation is going to be a collection of sets. This perspective comes very natural in a large number of application domains (graph problems deal with set of nodes/edges, planning problems deal with sets of actions or states, etc.).

Example 2. [27] The following simple ASP program computes the hamiltonian cycles of a graph.

```
%% Graph:
vertice(0). vertice(1). vertice(2). vertice(3).
edge(0,1). edge(0,2). edge(1,3). edge(1,2). edge(2,3). edge(3,0).
%% choice rules to select one edge
in(U,V) :- vertice(U), vertice(V), edge(U,V), not nin(U,V).
nin(U,V) :- vertice(U), vertice(V), edge(U,V), not in(U,V).
%% each node is traversed once
:- vertice(U), vertice(V), vertice(W), neq(V,W), in(U,V), in(U,W).
:- vertice(U), vertice(V), vertice(W), neq(U,V), in(U,W), in(V,W).

reachable(U) :- vertice(U), in(0,U).
reachable(V) :- vertice(U),vertice(V),reachable(U), in(U,V).
%% Guarantees that each vertex is reachable
:- vertice(U), not reachable(U).
```

For the graph in the example, the program admits a single answer set:

$$\{in(0,1), in(1,2), in(2,3), in(3,0)\}$$

In spite of these differences, ASP maintains many of the properties of LP, including its declarative nature, the separation between logic and control—where the logic is given by the content of the program and the control by the mechanisms used to compute the answer sets—and an underlying similar syntax.

2.3 Why ASP?

ASP has received great attention in knowledge representation and deductive databases, as it enables to represent default assumptions, constraints, uncertainty and nondeterminism *in a direct way* [5]. The automation of nonmonotonic reasoning may well rely upon automatic ASP, through the well-studied equivalences with, e.g., autoepistemic logic. ASP is related both ideally and through formal equivalences to the algorithmic study of satisfaction of boolean formulas (SAT). It is believed that ASP encoding of traditionally hard problems should be more compact than SAT encoding. For instance, [30] argues that ASP encodings of the Hamiltonian cycle problem are *asymptotically* more concise than SAT ones. This implies that, other things being equal, ASP interpretations can be as efficient as satisfiability and even as constraint satisfiability systems. For

example, [12] reports ASP solutions of planning problems in time comparable to ad-hoc planning algorithms. Finally, ASP syntax corresponds to $DATALOG^{\neg}$ of deductive databases, and should make database access transparent and straightforward.

3 Execution of ASP Programs

Computing with ASP is fairly different from computing in standard LP—in the latter we are interested in a single answer substitution computed w.r.t. a unique intended model, while in the former we are interested in computing sets of atoms representing different models. Designing an architecture for the computation of answer sets is not a straightforward task—indeed, the original definition of AS [20] is inherently non-constructive, as it requires guessing models and successively verifying whether they are answer sets or not. Nevertheless, in recent years a number of proposals have been made which provide approaches for computation of answer sets [7–9, 34, 17]. Chen & Warren [8] propose a method for computing answer sets which builds on their work on tabled evaluation of LP [39]. The method has the advantages of allowing a more relaxed program syntax and of being integrated in the context of an efficient Prolog system. On the other hand, the goal directed nature of this approach does not make it directly applicable as an engine for ASP [5]. Work is in progress by the XSB team to overcome this limitation. The three most efficient systems which support computation in the ASP paradigm are `dlv` [17], DeRes [9], and Smodels [34]. These systems, which have been proposed very recently and are continuously developing, provide comparable efficiency and relatively similar features. DeRes is a system originally developed to deal with a larger class of programs than ASP—*default theories*—but capable of efficiently handling ASP programs. The DeRes group currently provide a version of DeRes (called *stable*) which is specialized for the computation of answer sets, highly suitable as computational engine for ASP. `dlv` supports a very general language (which includes *disjunction*) and it provides different application specific front-ends—e.g., a front-end for abductive diagnosis [16]. This engine is very efficient—it is comparable or superior in speed to DeRes and smodels on various benchmarks [17]. The development of different implementations capable of handling ASP programs is very important—as it indicates the existence of a community which needs the power of ASP as well as it demonstrates that efficient execution of ASP is not beyond our reach.

3.1 Sequential Execution Models

The sequential architecture we propose in this project is a new generation engine obtained from the original design of the *Smodels* system [34]. Smodels relies on efficient and robust algorithms for the execution of ASP programs. The basic structure is sketched in Fig. 2—the algorithm alter-

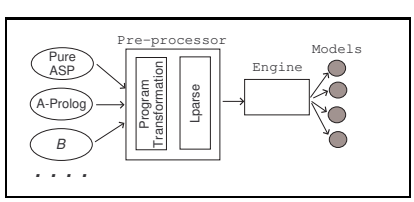

Fig. 1: Sequential Architecture

nates choices and propagations, in the style of typical constraint programming solutions. Figure 1 provides a schematic illustration of the overall proposed architecture. The components are described in this section.

Preprocessor Many of the systems proposed so far rely on the use of a preprocessor to transform the input program to a format suitable for processing. The preprocessor we propose includes the following components:

- *Program Transformation:* this is a collection of different modules used to perform source-to-source code transformation. Source transformations are used, first of all, to support alternative input languages which are mapped to the core ASP language [5, 6, 35]. Another objective of program transformations is to determine code transformations which can improve efficiency of execution.
- *Lparse:* answer sets semantics [20] relies on the manipulation of *ground programs*—i.e., programs which have been completely instantiated on a domain of interest and do not contain variables. During preprocessing it is necessary to *ground* the input program—identifying finite domains for each variable. Both smodels and DeRes rely on the same software, called *lparse* [46], which provides sophisticated grounding procedures.

Engine: The analogy between ASP and constraint programming, advocated by various researchers [30, 33] has been used to a certain extent in the design of existing ASP engines. Computation of answer sets relies on propagation techniques—selecting an atom as being true or false constrains, via the program rules, a number of other atoms to a specific logical value. Most systems take advantage of this feature (e.g., the `expand` procedure in Fig. 2). The proposal in [7] even translates the problem of computing answer sets into a problem of solving a linear programming problem—which can be tackled directly via constraint programming techniques. `choose_literal` selects one literal (i.e., an atom or its negation) to add to the answer set, while `expand` determines which atoms have a determined value in the partial answer set A. The actual algorithms used in many systems are refinements of this execution cycle.

The meaning of the partial answer set B is that, if atom a belongs to B, then a will belong to the final model. If *not* a belongs to B, a will *not* belong to the final model. Inconsistent interpretations are those containing contradictory atoms.

As ensues from Fig. 2, computation of answer sets is a highly non-deterministic and time-consuming activity. Non-determinism arises in different phases of this computation. The `expand` phase involves applying program rules in various ways (e.g., forward and backward chaining) to infer truth values of other literals. This process is seen as a fixpoint computation where, at each step, one rule is selected and used. Being the result of this phase deterministic, `expand` can be seen as an instance of *don't care non-determinism*. The fact that ASP programs may admit different answer sets implies that the `choose_literal` procedure is also non-deterministic; different choices will potentially lead to distinct answer sets. Thus, the process of selecting literals to add to the answer set represents a form of *don't know non-determinism*. This form of non-determinism has some resem-

blance to the non-determinism present in traditional LP (rule selection during one resolution step).

```
function compute (Π : Program, A : LiteralsSet)
    B := expand(Π, A) ;
    while ( (B is consistent) and
              (B is not complete) )
         l := choose_literal(Π, B);
         B := expand(Π , A ∪ { l }) ;
    endwhile
    if (B stable model of Π) then
         return B;
```

Fig. 2. Basic Execution Model for ASP

```
function expand (Π : Program, A : LiteralsSet)
    B := A ;
    while ( B ≠ B' ) do
         B' := B;
         B := apply_rule(Π, B);
    endwhile
    return B ;
```

Fig. 3. Expand procedure

Each non-deterministic computation can terminate either successfully—i.e., B assigns a truth value to all the atoms and it represents an answer set of Π—or unsuccessfully—if either the process tries to assigns two distinct truth values to the same atom or if B does not represent an answer set of the program (e.g., truth of certain selected atoms is not "supported" by the rules in the program). As in traditional logic programming, non-determinism is handled via backtracking to the choice points generated by `choose_literal`. Observe that each choice point produced by `choose_literal` has only two alternatives: one assigns the value true to the chosen literal, and one assigns the value false to it.

The `expand` procedure mentioned in the algorithm in Figure 2 is intuitively described in Figure 3. This procedure repeatedly applies expansion rules to the given set of literals until no more changes are possible. The expansion rules are derived from the program Π and allow to determine which literals have a definite truth value w.r.t. the existing partial answer set. This is accomplished by applying the rules of the program Π in different ways [34]. Efficient implementation of this procedure requires considerable care to avoid unnecessary steps, e.g., by dynamically removing invalid rules and by using smart heuristics in the `choose_literal` procedure [34], e.g.,

1. *forward rule:* if the rule $h \leftarrow a_1, \ldots, a_n, not\ b_1, \ldots, not\ b_m$ is in Π and all the elements in the body of the rule are true (i.e., they are in B), then also h can be assumed to be true in B
2. *nullary rule:* if there are no rules having the atom a as a head, then a can assumed to be false in B
3. *single positive rule:* if the atom h is true (i.e., it is in B) and there is a single rule

$$h \leftarrow a_1, \ldots, a_n, not\ b_1, \ldots, not\ b_m$$

in Π having h as head, then all the elements of the body of the rule can be added to B (i.e., they have to be true as well)

4. *negative rules:* if the literals *not* $h, l_1, \ldots, l_{i-1}, l_{i+1}, \ldots, l_m$ are in B and the rule

$$h \leftarrow l_1, \ldots, l_m$$

is in Π, then $\bar{l}_i$ can be added to B, where $\bar{l}$ indicates the complement of the literal l

Efficient implementation of this procedure requires considerable care to avoid unnecessary steps, e.g., by dynamically removing invalid rules and by using smart heuristics in the `choose_literal` procedure [45].

3.2 The NMSU System

The execution model sketched in the previous section has been originally proposed in the context of the *smodels* system, and adopted in a variety of other systems computing answer sets (e.g., some versions of the DLV system [19]). This execution model is at the core of the prototypes used for the investigation in parallel execution of ASP programs presented in this paper. The most complete system of these prototypes is called *jmodels* and it has been developed at NMSU. *jmodels* is a Java-based implementation of the propagation-based execution model for ASP computation, and it includes a number of advanced features, including:

- support for various language extensions, such as *choice rules* [46] and *weak constraints* [15];
- an object-oriented interface with Prolog, which allows Prolog programs to include ASP modules.

4 Sources of Parallelism

As described in Section 3.1, the execution model for ASP includes two major forms of non-determinism: a don't care choice in the selection of the rules during the expansion of a partial mode, and a don't know choice in the selection of a new (undefined) literal to be added to the current partial model. The presence of these two classes of non-determinism suggests a direction for *automatic parallelization* of the computation of answer sets.

5 Vertical Parallelism

The alternative choices of literals during the derivation of answer sets (`choose_literal` in Fig. 2) are *independent* and can be concurrently explored, generating separate threads of computation, each potentially leading to a distinct answer set. We will refer to this form of parallelism as *Vertical Parallelism*. Thus, *vertical parallelism* parallelizes the computation of *different* answer sets.

5.1 Issues in Managing Vertical Parallelism

As ensues from research on parallelization of search tree applications and non-deterministic programming languages [40, 2, 11, 23], the issue of designing the appropriate data structures to maintain the correct state in the different concurrent branches, is essential to achieve efficient parallel behavior. Observe that straightforward solutions to related problems have been formally proved to be ineffective, leading to unacceptable overheads [40].

The architecture for vertical parallel ASP that we envision is based on the use of a number of ASP engines (*agents*) which are concurrently exploring the search tree generated by the search for answer sets—specifically the search tree whose nodes are generated by the execution of the `choose_literal` procedure. Each agent explores a distinct branch of the tree; idle agents are allowed to acquire unexplored alternatives generated by other agents.

The major issue in the design of such architecture is to provide efficient mechanisms to support this sharing of unexplored alternatives between agents. Each node P of the tree is associated to a partial answer set $B(P)$—the partial answer set computed in the part of the branch preceding P. An agent acquiring an unexplored alternative from P needs to continue the execution by expanding $B(P)$ together with the literal selected by `choose_literal` in node P. Efficient computation of $B(P)$ for the different nodes in the tree is a known complex problem [40].

Since ASP computations can be very ill-balanced and irregular, we opt to adopt a dynamic scheduling scheme, where idle agents navigate through the system in search of available tasks. Thus, the partitioning of the available tasks between agents is performed dynamically and is initiated by the idle agents. This justifies the choice of a design where different agents are capable of traversing a shared representation of the search tree to detect and acquire unexplored alternatives. In addition, this view allows one to reuse the *optimization schemes* developed i for other parallel execution models to improve efficiency of these mechanisms, via run-time transformations of the search tree [22]—e.g., flattening the tree to facilitate work sharing.

Basic Structure of the Parallel Engine As mentioned earlier, the system is organized as a collection of agents which are cooperating in computing the answer sets of a program. Each agent is a separate ASP engine, which owns a set of *private* data structures employed for the computation of answer sets. Additionally, a number of *global* data structures, i.e., accessible by all the agents, are introduced to support cooperation between agents. This structuring of the system implies that we rely on a shared-memory architecture.

The different agents share a common representation of the ASP program to be executed. This representation is stored in one of the global data structures. Program representation has been implemented following the general data structure originally proposed in [13]—proved to guarantee very efficient computation of standard models. This representation is summarized in Figure 4. Each rules

is represented by a descriptor; all rules descriptors are collected in a single array, which allows for fast scan of the set of rules. Each rule descriptor contains, between the other things, pointers to the descriptors for all atoms which appear in the rule—the head atom, the atoms which appear positive in the body of the rule, and the atoms which appear negated in the body of the rule.

Each atom descriptor contains information such as

- an array containing pointers to the rules in which the atom appears as head
- an array containing pointers to the rules in which the atom appears as positive body element
- an array containing pointers to the rules in which the atom appears as negative body element
- an *atom array* index

Differently from the schemes adopted in sequential ASP engines [13, 34], our atom descriptors *do not* contain the truth value of the atom. Truth values of atoms are instead stored in a separate data structure, called *atom array.* Each agent maintains a separate atom array, as shown in Figure 4; this allows each agent to have an independent view of the current (partial) answer set constructed, allowing atoms to have different truth values in different agents. E.g., in Figure 4, the atom of index i is true in the answer set of one agent, and false in the answer set computed by another agent.

Each agent essentially acts as a separate ASP engine. Each agent maintains a local stack structure (the *trail*) which keeps track of the atoms whose truth value has already been determined. Each time the truth value of an atom is determined (i.e., the appropriate entry in the atom array is set to store the atom's truth value), a pointer to the atom's descriptor is pushed in the trail stack. The trail stack is used for two purposes:

- (during `expand`) the agent uses the elements newly placed on the trail to determine which program rules may be triggered for execution;
- a simple test on the current size of the trail stack allows each agent to determine whether all atoms have been assigned a truth value or not.

The use of a trail structure provides also convenient support for exploitation of horizontal parallelism [18].

To support the exploitation of vertical parallelism, we have also introduced an additional simple data structure: a *choice point* stack (or *core stack*). The elements of the choice point stack are pointers to the trail stack. These pointers are used to identify those atoms whose truth value has been "guessed" by the `choose_literal` function. The choice points are used during backtracking: they are used to determine which atoms should be removed from the answer set during backtracking, as well as which alternatives can be explored to compute other answer sets. This is akin to the mechanisms used to support backtracking in trail-based constraint systems [42, 43].

The open issue which remains to be discussed is *how* agents *interact* in order to exchange unexplored alternatives—i.e., how agents *share* work. Each

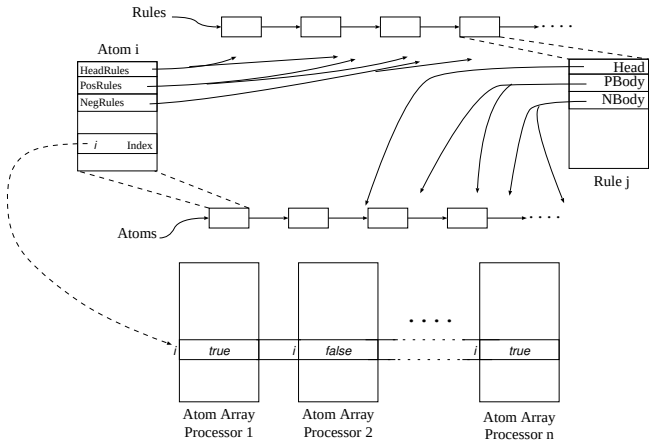

Fig. 4. Representation of Rules and Atoms

idle agent attempts to obtain unexplored alternatives from other active agents. In our context, an *unexplored alternative* is represented by a partial answer set together with a new literal to be added to it.

In this project we have explored two alternative approaches to tackle this problem:

- *Recomputation-based Work Sharing:* agents share work just by exchanging the list of *chosen* literals which had been used in the construction of an answer set; the receiving agent will use these to *reconstruct* the answer set and then perform local backtracking to explore a new alternative.
- *Copy-based Work Sharing:* agents share work by exchanging a complete copy of the current answer set (both *chosen* as well as *determined* literals) and then performing local backtracking.

The two schemes provide a different balance between amount of data copied from one agent to the other and amount of time needed to restart the computation with a new alternative in a different agent. These two methods are discussed in detail in the next sections. Although many alternative methods have been discussed in the literature to handle parallel execution of search-based applications (e.g., see [21] for a survey), we have focused on these two models for the following reasons:

- these two methodologies have been theoretically demonstrated to be *optimal* with respect to a reasonable abstraction of the problem of supporting concurrent search;
- our intention is to target exploitation of vertical parallelism across a wide range of parallel platforms, including distributed memory platforms. It has been proved that these two methodologies are the most effective in absence of shared memory.

Another important aspect that has to be considered in dealing with this sort of systems is termination detection. The overall computation needs to determine when a global fixpoint has been reached—i.e., all the answer sets have been produced and no agent is performing active computation any longer. In the system proposed we have adopted a centralized termination detection algorithm. One of the agents plays the role of controller and at given intervals polls the other agents to verify global termination. Details of this algorithm are omitted for lack of space.

Model Recomputation The idea of recomputation-based sharing of work is derived by similar schemas adopted in the context of *or-parallel* execution of Prolog [23]. In the recomputation-based scheme, an idle agent obtains a partial answer set from another agent in an *implicit* fashion. Let us assume that agent $\mathcal{A}$ wants to send its partial answer set B to agent $\mathcal{B}$. To avoid copying the whole partial answer set B, the agents exchange only a list containing the literals which have been chosen by $\mathcal{A}$ during the construction of B. These literals represent the "core" of the partial answer set. In particular, we are guaranteed that an `expand` operation applied to this list of literals will correctly produce the whole partial answer set B. This communication process is illustrated in Fig. 5. The core of the current answer set is represented by the set of literals which are pointed to by the choice points in the core stack (see Fig. 5). In particular, to make the process of sharing work more efficient, we have modified the core stack so that each choice point not only points to the trail, but also contains the corresponding chosen literal (the literal it is pointing to in the trail stack). As a result, when sharing of work takes place between agent $\mathcal{A}$ and agent $\mathcal{B}$, the only required activity is to transfer the content of the core stack from $\mathcal{A}$ to $\mathcal{B}$. Once $\mathcal{B}$ receives the chosen literals, it will proceed to install their truth values (by recording the literals' truth values in the Atom Array) and perform an `expand` operation to reconstruct (on the trail stack) the partial answer set. The last chosen literal will be automatically complemented to obtain the effect of backtracking and constructing the "next" answer set. This copying process can be also made more efficient by making it *incremental*: agents exchange only the *difference* between the content of their core stacks. This reduces the amount of data exchanged and allows to reuse part of the partial answer set already existing in the idle agent.

Model Copying The copying-based approach to work sharing adopts a simpler approach then recomputation. Upon work sharing from agent $\mathcal{A}$ to $\mathcal{B}$, the entire partial answer set existing in $\mathcal{A}$ is directly copied to agent $\mathcal{B}$. The use of copying has been frequently adopted to support computation in constraint programming systems [43] as well as to support or-parallel execution of logic and constraint programs [23]. The partial answer set owned by $\mathcal{A}$ has an explicit representation within the agent $\mathcal{A}$: it is completely described by the content of the trail stack. Thus, copying the partial answer set from $\mathcal{A}$ to $\mathcal{B}$ can be simply reduced to the copying of the trail stack of $\mathcal{A}$ to $\mathcal{B}$. This is illustrated in Figure 6. Once this copying has been completed, $\mathcal{B}$ needs to install the truth value of

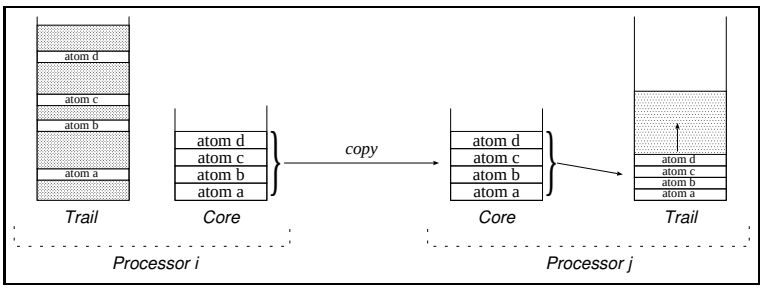

Fig. 5. Recomputation Sharing of Work

the atoms in the partial answer set—i.e., store the correct truth values in the atom array. Computation of the "next" answer set is obtained by identifying the most recently literal whose value has been "guessed" and performing local backtracking to it. The identification of the backtracking literal is immediate as this literal lies always at the top of copied trail stack. As in the recomputation case, we can improve performance by performing incremental copying, i.e., by copying not the complete answer set but only the difference between the answer set in $\mathcal{A}$ and the one in $\mathcal{B}$.

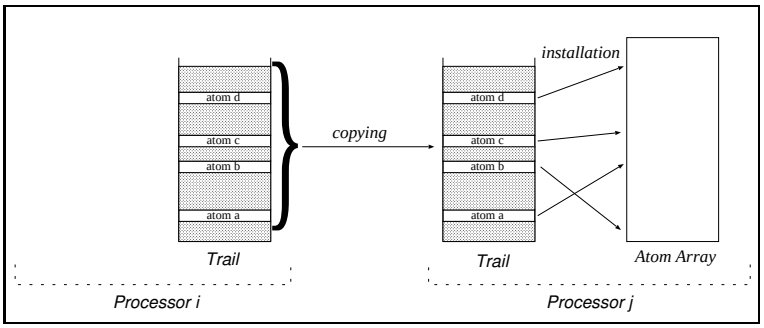

Fig. 6. Copy-based Sharing of Work

Hybrid Sharing Schemes The experiments performed on shared memory architectures (described in the next Section) have indicated that Model Copying behaves better than Model Recomputation in most of the cases. This is due to the high cost of recomputing parts of the answer set w.r.t. the cost of simply performing a memory copying operation. This property does not necessarily hold any longer when we move to distributed memory architectures (as the Beowulf platform used in this project), due to the considerably higher cost for copying data between agents.

To capture the best of both worlds, we have switched in our prototype to an hybrid work sharing scheme, where both Model Recomputation and Model Copying are employed. The choice of which method to use is performed dynamically (*each time* a sharing operation is required). Various heuristics have been considered for this selection, which take into account the size of the core and the size of the partial answer set. Some typical observations that have been made from our experiments include: *(i)* if the size of the core is sufficiently close to the size of the answer set, then recomputation would lead to a loss w.r.t. copying. *(ii)* if the size of the answer set is very large compared to the size of the core, then copying appears still to be more advantageous than recomputation. This last property is strongly related to the speed of the underlying interconnection network—the slower the interconnection network, the larger is the partial answer set that one can effectively recompute. We have concretized these observations by experimentally identifying two thresholds (*low* and *high*) and a function f which relates the size of the core and the size of the answer set; Recomputation is employed whenever $low \leq f(\, sizeof(Core), sizeof(Partial\ Answer\ Set)\,) \leq high$. The same considerations are even more significant in the context of execution of ASP on distributed memory platforms: in this context the cost of copying is higher (due to the higher cost of moving data across the interconnection network) and the threshold in favor of recomputation is wider.

5.2 Experimental Results

Model Recomputation on Shared Memory Platforms In this section we present performance results for a preliminary prototype which implements an ASP engine with Recomputation-based vertical parallelism. The prototype used has been developed in C and the performance results have been obtained on a 14-processor Sun Enterprise. The prototype is capable of computing the answer sets of standard ASP programs, pre-processed by the lparse grounding program [46]. The prototype is largely unoptimized (e.g., it does not include many of the heuristics adopted in similar ASP engines [45]) but its sequential speed is reasonably close to that of the efficient Smodels system[4] [34].

All performance figures presented are in milliseconds and have been achieved as average execution times over 10 consecutive runs on a very lightly loaded machine. The benchmarks adopted are programs obtained from various sources (all written by other researchers); they include some large scheduling applications (sjss, rcps), planners (logistics 1,2, strategic), graph problems (color), as well various synthetic benchmarks (T4, T5, T15, T8, P7). These benchmarks range in size from few tens of rules (e.g., T4, T5) to hundreds of rules (e.g., rcps).

As can be seen from the figures in Table 1, the system is capable of producing good speedups from most of the selected benchmarks. On the scheduling (sjss, rcps), graph coloring, and planning (strategic, logistics) benchmarks the speedups are very high (mostly between 6 and 8 using 10 agents). This is quite a remarkable result, considering that these benchmarks are very large and some

[4] Comparisons made with the lookahead feature of Smodels turned off.

Name	1 Agent	2 Agents	3 Agents	4 Agents	8 Agents	10 Agents
Scheduling (sjss)	131823.88	66146.75	44536.16	34740.25	19132.63	16214.49
Scheduling (rcps)	72868.48	36436.24	28923.33	18040.35	13169.61	10859.68
Color (Random)	1198917.24	599658.62	389737.88	300729.31	178942.87	158796.98
Color (Ladder)	1092.81	610.73	441.61	353.80	325.00	306.67
Logistics (1)	10054.68	10053.78	10054.71	4545.31	3695.67	3295.23
Logistics (2)	6024.67	3340.44	2763.14	2380.36	1407.53	1371.47
Strategic	13783.51	7317.02	5018.43	4005.83	2278.18	1992.51
T5	128.21	67.32	69.97	72.11	76.55	77.62
T4	103.01	78.14	78.33	84.11	91.79	118.21
T8	3234.69	1679.29	1164.25	905.21	761.69	710.37
P7	3159.11	1679.86	1266.91	981.75	445.33	452.19
T15	415.73	221.70	178.94	132.10	135.99	137.11
T23	3844.41	1991.76	1595.75	1433.56	1341.79	1431.70

Table 1. Recomputation-based Sharing: Execution Times (msec.)

produce highly unbalanced computation trees, with tasks having very different sizes. The apparently low speedup observed on the logistics with the first plan (logistics 1), is actually still a positive result, since the number of choices performed across the computation is just 4 (thus we cannot expect a speedup higher than 4). On the very fine-grained benchmarks T4 and T5 the system does not behave as well; in particular we can observe a degradation of speedup for a large number of agents—in this case the increased number of interactions between agents overcome the advantages of parallelization, as the different agents attempt to exchange very small tasks. In T4 we even observe a slow-down when using more than 8 agents. Two slightly disappointing results are in T8 and P7. T8 is a benchmark which produces a very large number of average-to-small size tasks; the top speedup is below 5 and denotes some difficulty in maintaining good efficiency in presence of frequent task switching. P7 on the other hand has a very low number of task switching, but generates extremely large answer sets. The speedup tends to decrease with large number of agents because some agents end up obtaining choice points created very late in the computation, and thus waste considerable time in rebuilding large answer sets during the recomputation phase. The speedups obtained for all these benchmarks are plotted in Figure 7.

Note that the sequential overhead observed in all cases (the ratio between the sequential engine and the parallel engine running on a single processor) is extremely low, i.e., within 5% for most of the benchmarks.

Model Copying on Shared Memory Platforms We have modified our implementation to support Copy-based work sharing, and we have tested its

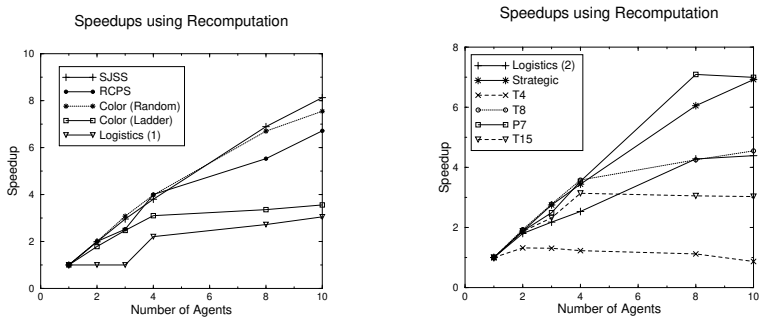

Fig. 7. Speedups using Recomputation

performance on the same pool of benchmarks. Also in this case, the sequential overhead is very low (on average it is below 5%).

Name	**1 Agent**	**2 Agents**	**3 Agents**	**4 Agents**	**8 Agents**	**10 Agents**
Scheduling (sjss)	131823.89	65911.94	44236.20	33038.57	18182.61	15187.08
Scheduling (rcps)	72868.49	36434.25	22421.09	18217.13	9794.15	6854.99
Color (Random)	1198917.31	599458.66	355761.84	282763.52	164010.55	135013.21
Color (Ladder)	1092.80	590.70	401.77	337.28	287.58	286.07
Logistics (1)	10054.70	4429.38	4224.67	4206.99	2739.70	2674.12
Logistics (2)	6024.67	3089.57	2619.43	2042.26	1149.75	1079.69
Strategic	13662.61	7273.10	5043.04	3534.24	2015.14	1832.87
T5	81.60	42.70	43.90	44.10	48.53	51.14
T4	98.20	61.42	54.01	65.47	65.06	65.96
T8	3232.58	1602.89	1086.90	798.38	437.77	362.49
P7	3160.00	1847.95	1392.07	1078.49	556.34	497.64
T15	416.00	208.01	138.67	118.17	120.23	122.35
T23	2695.81	1395.12	1017.61	769.31	402.55	464.18

Table 2. Copy-based Sharing: Execution Times (msec.)

The results reported in Table 2 (and the corresponding speedups plotted in Figure 8) are remarkable. The large benchmarks (e.g., the two scheduling applications) report speedups in the range $8.5 - 10$ for 10 agents, maintaining linear speedups for small number of agents (from 2 to 5 agents).

The fine grained benchmarks (such as T4 and T5) provide speedups similar (usually slightly better) to those observed earlier. In both cases we note a slight

degradation of speedup for large number of agents. As in the case of recomputation, this indicates that if the tasks are too fine grained, additional steps are needed in order to achieve performance improvements. We have experimented with a simple optimization, which semi-automatically unfolds selected predicates a constant number of times, in order to create larger grain tasks (by effectively combining together consecutive tasks). The simple optimization has produced improvements, as show in Table 3.

Name	1 Agent	2 Agents	3 Agents	4 Agents	8 Agents	10 Agents
T5	1.0	1.99/1.91	1.99/1.86	1.97/1.85	1.95/1.68	1.93/1.60
T4	1.0	1.92/1.60	1.93/1.82	1.95/1.50	1.93/1.51	1.91/1.49

Table 3. Speedup Improvement using Task-collapsing Optimization (new/old)

The Copy-based scheme behaves quite well in presence of a large number of average-to-small tasks, as seen in the T8 benchmark. The speedups reported in this case are excellent. This is partly due to the lower cost, in this particular case, of copying w.r.t. recomputation, as well as the adoption of a smarter scheduling strategy, made possible by the use of copying, as discussed in the next section.

For what concerns the benchmark P7, the situation is sub-optimal. In this case the need of copying large answer sets during sharing operations penalizes the overall performance. We expect this case to become less of a problem with the introduction of *incremental copying* techniques—i.e., instead of copying the whole answer set, the agents compute the actual difference between the answer sets currently present in their stacks, and transfer only such difference. Our current prototype does not include this optimization.

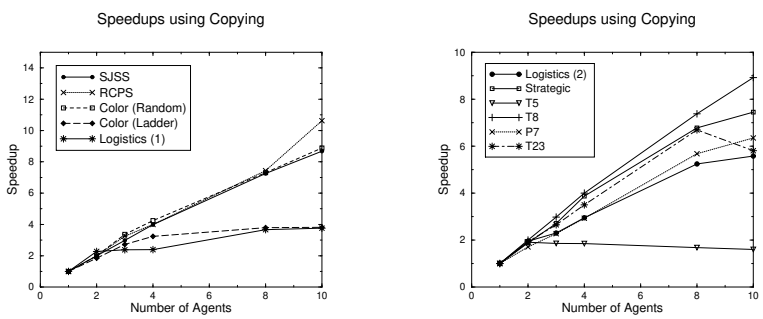

Fig. 8. Speedups using Copying

Performance on Distributed Memory Platforms The engine used in the previous experiment has been converted to support execution on distributed platforms—simply converting memory copying operation to message passing (based on MPI). The results have been obtained on the Pentium-based Beowulf (purely distributed memory architectures) at NMSU—Pentium II (333Mhz) connected via Myrinet. The results reported have been obtained from two similar implementations of ASP, one developed at NMSU and one at TTU. Both systems have been constructed in C using MPI for dealing with interprocessor communication. The experiments have been performed by executing a number of ASP programs (mostly obtained from other researchers) and the major objective was to validate the feasibility of parallel execution of ASP programs on Beowulf platforms.

All timings presented have been obtained as average over 10 runs. As mentioned in Sect. 5.1, in our design we have decided to adopt an Hybrid Method to support exchange of unexplored tasks between agents. This is different from what we have observed in the case of shared memory execution, where Model Copying was observed to be the winning strategy in the last majority of the benchmarks. In the context of distributed memory architectures, the higher cost of communication between processors leads to a higher number of situations where the model copying provides sub-optimal performances.

Table 4 reports the execution times observed on a set of benchmarks, while Fig. 9 illustrates the speedups observed using the hybrid scheme on a set of ASP benchmarks. Some of the benchmarks, e.g., `T8` and `P7`, are synthetic benchmarks developed to study specific properties of the inference engine, while others are ASP programs obtained from other researchers. `Color` is a graph coloring problem, `Logistics` and `Strategic` are scheduling problems, while `sjss` is a planner. Note also that `sjss` is executed searching for a single model while all others are executed requiring all models to be produced. The tests marked `[*]` in Fig. 9 indicate those cases where Recomputation instead of Copying has been triggered the majority of the times. The results presented have been accomplished by using an experimentally determined threshold to discriminate between copying and recomputation. The rule adopted in the implementation can be summarized as: if

$$min \leq \frac{size(Partial\ Answer\ Set)}{size(Core)} \leq max$$

then model recomputation is applied, otherwise model copying is used. The intuition is that *(i)* if the ratio is too low, then, there is no advantage in copying just the core, while *(ii)* if the ratio is too high, then the cost of recomputing the answer set is likely to be excessive. The *min* and *max* used for these experiments where set to 1.75 and 12.5. Fig. 10 shows the impact of using recomputation in the benchmarks marked with `[*]` in Fig. 9. Some benchmarks have shown rather low speedups—e.g., `Color` on a ladder graph and `Logistics`. The first generates very fine grained tasks and suffers the penalty of the cost of communication between processors—the same benchmarks on a shared-memory platform produces speedups close to 4. For what concerns `Logistics`, the results are, after all, quite

positive, as the maximum speedup possible is actually 5 and there seem to be no degradation of performance when the number of agents is increased beyond 5.

Name	1 Agent	2 Agents	3 Agents	4 Agents	8 Agents
Color (Ladder)	345201	249911	235421	292932	295420
Color (Random2)	2067987	1162905	829685	604586	310622
Logistics 2	3937246	2172124	1842695	1652869	1041534
Strategic	76207	40169	28327	21664	12580
sjss	93347226	46761140	31012367	22963465	13297326
T8	1770106	865175	590035	444730	226930
P7	1728001	918172	690924	536646	216040

Table 4. Execution Times (in μs.) on Beowulf

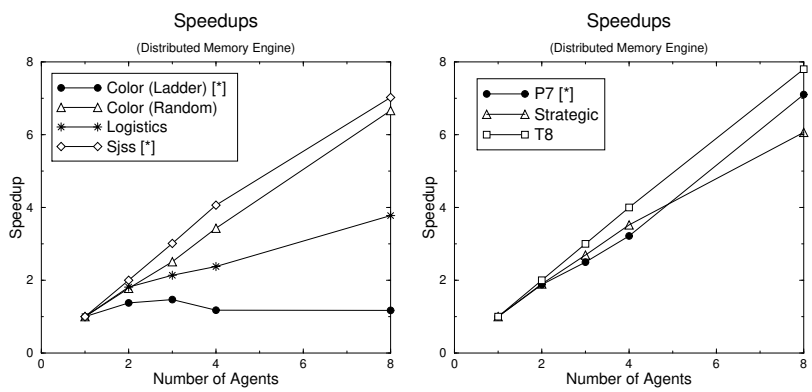

Fig. 9. Speedups from Vertical Parallelism

It is interesting to compare the behavior of the distributed memory implementation with that of the shared memory engine presented in the previous subsection. Fig. 11 presents a comparison between the speedups observed on selected benchmarks in the shared memory and the distributed memory engines. In the majority of the cases we observed relatively small degradation in the speedup. Only benchmarks where frequent scheduling of small size tasks is required lead to a more relevant difference (e.g., `Color` for the ladder graph).

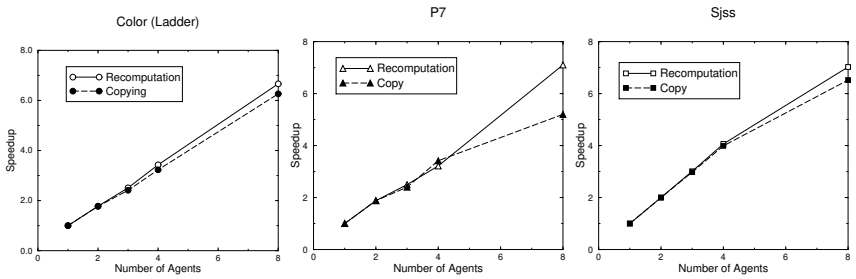

Fig. 10. Impact of using Recomputation

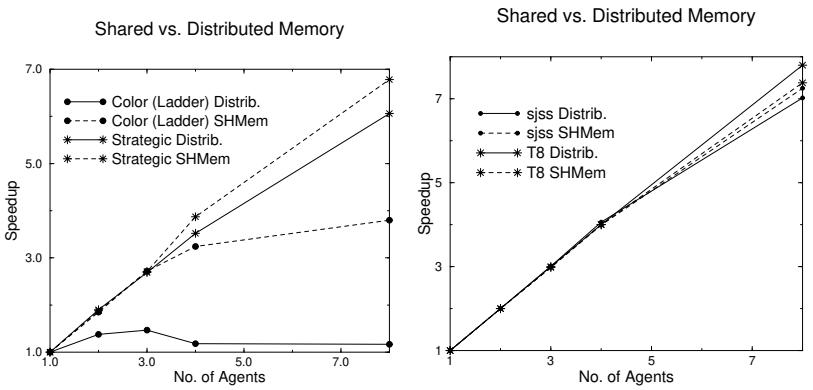

Fig. 11. Comparison of Shared and Distributed Memory Engines

5.3 Scheduling Vertical Parallelism

In the context of our system, two scheduling decisions have to be taken by each idle processor in search of work:

1. select from which agent work will be taken;
2. select which unexplored alternative will be taken from the selected agent.

In the current prototype, we have tackled the first issue simply by maintaining a *work-load* count (i.e., number of local unexplored alternatives) for each agent and attempting to take work from the agent with the highest work-load. This simple scheme has proved to work well in practice.

The second decision turned out to be more complicated and has a deeper impact on the performance of the system. Our experimental results have indicated that the choice of which unexplored alternative to take work from (i.e., which choice point to steal from another agent) may lead to substantial variations in parallel performance.

In our experiments we have considered two approaches to this problem. In the first approach, agents are forced to steal the first choice point (i.e., the oldest

choice point) from another agent (we call this approach *Top scheduling*). This technique was expected to perform well since:

- detecting the first choice point is a fast operation;
- selecting the first choice point reduces the size of the partial answer set transfered between agents;
- if the computation tree is balanced, then by taking the first choice point we should minimize the frequency of sharing operations.

The alternative technique considered is the dual of the one described above: at each sharing operation the last choice point created is taken (we call this approach *Bottom scheduling*). This approach is expected to have the following advantage: with simple modifications to the backtracking scheme, it allows to share at once not just a single choice point but a collection of them—e.g., all the choice points owned by an agent. On the other hand, the cost of sharing work under this scheme is considerable higher, since larger answer sets have to be exchanged.

The implementation of the first method is relatively simple; the first choice point is easily detected (by keeping an additional register in each agent for this purpose). The choice point indicates the segment of trail that has to be transferred to the other agent.

The second method has been realized as follows:

- the last choice point is easily detected as it lies on the top of the choice point stack; this allows to determine immediately what is the part of the trail that has to be copied;
- to allow sharing of multiple choice points at once, we push on the choice point stack a special choice point, which simply represents a link to a choice point lying in another processor's stack. This allows the backtracking activity to seamlessly flow between choice points belonging to different agents. (This technique resembles a similar methodology used for *public backtracking* in and-parallel logic programming systems [37]).

We have implemented both schemes and compared them on the selected pool of benchmarks. Figure 12 compares the speedups achieved using the two scheduling schemes in the Copy-based sharing scheme. The results clearly indicate that the second scheme (Bottom scheduling) is superior in the large majority of the cases. Particularly significant are the differences in the sjss and the graph coloring problems. These are all programs where a large number of choices points are created; the bottom scheduling scheme allows to share in a single sharing operation a large number of alternatives, thus reducing the number of scheduling interactions between agents. The Top scheduling scheme provides better performance in those benchmarks where either there are few choices (e.g., T15) or the choice tend to be located always towards the beginning of the trail stack (T8).

Also in this case we can clearly identify a preferable scheme (the Bottom scheduling scheme); nevertheless a mixed approach which selects alternative scheduling policies depending on the structure of the program or the structure of the current answer set is likely to provide superior performance.

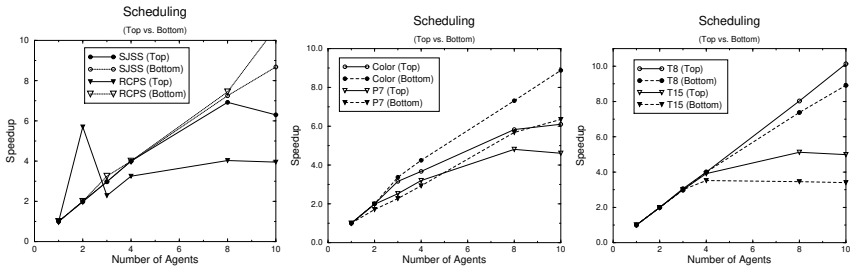

Fig. 12. Scheduling: Top vs. Bottom Scheduling

5.4 Optimizing Vertical Parallelism

We have also explored the performance of the distributed engine on a number of other benchmarks. The preliminary results obtained on this second batch of benchmarks were rather disappointing; indeed, on a number of sufficiently large grain computations, we observed severe slow-downs when increasing the number of agents employed. The problem was pinpointed to derive from the large size of the models generated by these benchmarks. During the sharing operations, each idle agent has to undo the existing computation (via backtracking), receive a complete copy of the trail, and install the new entries. As pointed out earlier, the current prototype does not include the notion of incremental copying, which may improve this sort of situations. Instead of building the complete infrastructure for incremental copying—which is potentially quite complex—we have tried to simply optimize the task of abandoning the current computation. Instead of blindly proceeding in a complete backtracking phase, the idle processor performs a test on the current size of the partial model located in its trail stack. If the size is above an experimentally determined threshold, then complete backtracking is replaced by a brute-force memory zeroing operation (using Unix's `memset`) to wipe out the content of the atom array. Experimental results have shown that if the trail's content is very large, this operation is considerably faster.

Figure 13 compares the speedup curves achieved with and without this optimization. While for the benchmarks in the left diagram the improvements are relatively small, the benchmarks on the right indicate that the impact of this optimization can be very high. Both benchmarks (`Color 6` is the coloring of another large ladder graph, while `rpcs 4` is a different version of the `rpcs` scheduling program) lead to a slowdown using a large number of agents. The use of the optimization allows the benchmarks to produce acceptable (for the `Color 6` program) or really good (for the `rpcs 4` benchmark) speedups. It is important to observe that the cost of the optimization is negligible (a simple test), compared to the cost of performing a full-blown incremental copying (e.g., cost of determining the part of the trail in common between two interacting agents).

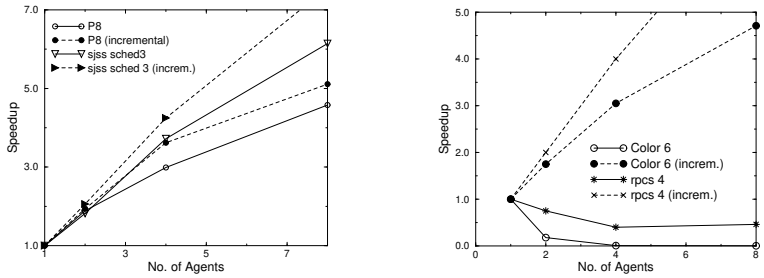

Fig. 13. Speedup Curves with and without Memory Zeroing Optimization

6 Horizontal Parallelism

An orthogonal direction for parallel ASP can be achieved by parallelizing the steps occurring along *one* branch of the search tree. This implies, as in Fig. 2, the parallel evaluation of the individual executions of `expand`. `expand` determines what are the literals whose truth value is immediately determined by the partial answer set B. This is achieved by applying in various ways the program rules (e.g., forward and backward chaining), to expand the partial answer set, without performing any choice. Each rule can provide a different contribution, depending on the partial answer set. *Horizontal parallelism* can be achieved by allowing concurrent application of different rules to expand the partial answer set.

6.1 Static Horizontal Parallelism

The most direct approach in the exploitation of horizontal parallelism arises from the parallelization of the operations present in the `expand` procedure. As illustrated in Section 3.1, the `expand` operation consists of a fixpoint computation aimed at expanding the partial answer set by applying in different ways the rules present in the program.

Static parallelization of this process is obtained by partitioning the set of program rules between the different processors, so that each processor is in charge of applying a given set of rules (*program fragment*) to the partial answer set.

Two major issues have to be considered when developing an horizontal parallel ASP solution:

1. *partitioning scheme:* the partitioning scheme is the policy used to distribute the program rules between the set of available processors.
2. *interaction policy:* the interaction policy determines the frequency and pattern of interaction between the processors cooperating in the expansion of a partial answer set.

In our preliminary experiments we have adopted the following policies:

1. *partitioning scheme:* the current policy is derived from the work on parallel constraint propagation [32] and assigns to each processor a collection of *procedures*—where a procedure is a collection of all the rules with the same predicate in the head. The partitioning is *static* and it is not modified during the execution of the program. The collection of procedures assigned to the processors are determined according to two heuristics:
   - processors should receive fragments of the same size—where the size is given by the total number of atoms in the fragment;
   - procedures which are "close" in the dependence graph of the ASP program are assigned to the same processor. In particular, the current heuristics tries to keep elements belonging to a strongly connected component of the dependence graph within the same and-agent.
2. *interaction policy:* each and-agent maintains a local stack where it maintains the part of the partial answer set that has been determined exclusively using the locally available program rules. Each time a local fixpoint is determined, the content of the local stack is transferred to a global representation (in shared memory) of the partial answer set.

We have initiated the development of an horizontal parallel ASP engine constructed according to the previously described policies. The preliminary prototype has been developed using Posix Threads and tested on a Pentium-based shared-memory platform with 4-processors (200Mhz Pentium-Pro, running Solaris 8). The prototype is still under completion, and it has been tested only on a collection of automatically generated synthetic benchmarks (`Synth1` through `Synth4`). The benchmarks are composed of 50,000 program rules, each having a random number of body elements (between 0 and 4). The results are presented in Table 5.

Benchmark	**Number of Agents**			
	1	**2**	**3**	**4**
`Synth1`	393662	37945	17720	2048
`Synth2`	811335	89353	2337	1852
`Synth3`	2565765	132473	61480	3426
`Synth4`	64386763	260800	211890	45162

Table 5. Execution Times (in μsec)

The current prototype has the following properties:

- the indicated benchmarks show super-linear speedups; this arises from the fact that all the synthetic benchmarks considered do not have models—the parallel execution allows to detect inconsistencies faster.
- it provides sub-optimal sequential performance, due to excessive locking in the access of the shared representation of the partial answer set.

Wok is in progress to attempt to reduce communication costs and sustain acceptable speedups on regular (non-contradictory) benchmarks.

6.2 Lookahead Parallelism

The (sequential) *smodels* algorithm presented earlier builds the stable models of an answer set program incrementally. The algorithm presented in Fig. 2 can be refined to introduce the use of lookahead during the "guess" of a literal. The algorithm is modified as follows: *(1)* Before guessing a literal to continue expansion, unexplored literals are tested to verify whether there is a literal l such that $\mathsf{expand}(\Pi, B \cup \{l\})$ is consistent and $\mathsf{expand}(\Pi, B \cup \{not\, l\})$ is inconsistent. Such literals can be immediately added to B. *(2)* After such literals have been found, choose_literal can proceed by guessing an arbitrary unexplored literal. Step 1 is called the `lookahead` step. It is important to observe that any introduction of literals performed in this step is *deterministic* and does not require the creation of a choice point. In addition, the work performed while testing for the various unexplored literals can be used to choose the "best" literal to be used in step 2, according to some heuristic function.

During the lookahead step, every test performed on a pair $\langle l, not\ \ l\rangle$ is substantially independent from the tests run on any other pair $\langle l', not\ \ l'\rangle$. Each test involves up to two calls to expand (one for a, the other one for $not\ \ a$), thus resulting in a comparatively expensive computation. These characteristics make the lookahead step a natural point where the algorithm could be parallelized. Notice that *Parallel Lookahead* is an instance of the general concept of Horizontal Parallelism, since the results of the parallel execution of lookahead are combined, rather than being considered alternative to each other, as in Vertical Parallelism. The appeal of exploiting Horizontal Parallelism at the level of `lookahead`, rather than at the level of `expand`, lies in the fact that the first involves a coarser-grained type of parallelism.

Basic Design The parallelization of the lookahead step is obtained in a quite straightforward way by splitting the set of unexplored literals, and assigning each subset to a different agent. Each agent then performs the test described in step 1 on the unexplored literals that it has been assigned. Finally, a new partial answer set, B' is built by merging the results generated by the agents. Work sharing is the Model Copying technique.

Notice that, even in the parallel implementation, the lookahead step can be exploited in order to determine the best literal to be used in `choose_literal` (provided that the results returned by the agents are suitably combined). This greatly reduces the computation performed by `choose_literal`, and provides a simple way of combining Vertical and Horizontal Parallelism by applying a work-sharing method similar to the *Basic Andorra Model* [23], studied for parallelization of Prolog computation.

Scheduling The key for the integration of Vertical and Horizontal Parallelism is in the way work is divided in work units and assigned to the agents. Our system is based on a central scheduler, and a set of agents that are dedicated to the actual computation of the answer sets. Every work unit corresponds to a lookahead step performed on a partial answer set, B, using a set of unexplored literals, U. Work units related to different partial answer sets can be processed at the same time by the system. Every time all the work units associated with certain partial answer set have been completed, the scheduler gathers the results and executes `choose_literal` – which, as we stated before, requires a very small amount of computation, and can thus be executed directly on the scheduler. `choose_literal` returns two (possibly) partial answer sets[5], and the scheduler generates work units for both of them, thus completing a (parallel) iteration of the algorithm in Fig. 2, extended with `lookahead`. Under this perspective, Horizontal Parallelism corresponds to the parallel execution of work units related to the same partial answer set. Vertical Parallelism, instead, is the parallel execution of work units related to different partial answer sets. The way the search space is traversed, as well as the balance between Vertical and Horizontal Parallelism, are determined by: *(1)* the number agents among which the set of unexplored literals is split, and *(2)* the priority given to pending work units. In our implementation we assign priorities to pending work units according to a "simulated depth first" strategy, i.e., the priority of a work unit depends first on the depth, d, in the search space, of the corresponding node, n, and second on the number of nodes of depth d present to the left of n. This choice guarantees that, if a computation based only on Horizontal Parallelism is selected, the order in which nodes are considered is the same as in a sequential implementation of our algorithm.

The number of agents among which the set of unexplored literals is split is selected at run-time. This allows the user to decide between a computation based on Horizontal Parallelism, useful if the answer set(s) are expected to be found with little backtracking, and a computation based on Vertical Parallelism, useful if a more backtracking is expected.

Experimental Results The experiments on Parallel Lookahead have been conducted using the distributed ASP engine developed at TTU. For our tests, we have used a subset of the benchmarks available at `http://www.tcs.hut.fi/pub/smodels/tests/lp-csp-tests.tar.gz`: *(1)* `color`: c-colorability (4 colors, 300 nodes), *(2)* `pigeon`: put N pigeons in M holes with at most one pigeon in a hole ($N = 24, M = 24$), *(3)* `queens`: N-queens problem ($N = 14$), and *(4)* `schur`: put N items in B boxes such that, for any $X, Y \in \{1, \ldots, N\}$: items labeled X and $2X$ are in different boxes, and if X and Y are in the same box, then $X + Y$ is in a different box ($N = 35, B = 15$).

The tests consisted in finding one answer set for each of these programs. Since, for all of these programs, this can be accomplished with a comparatively

[5] Our version of `choose_literal` runs `expand` on the two partial answer sets before returning them.

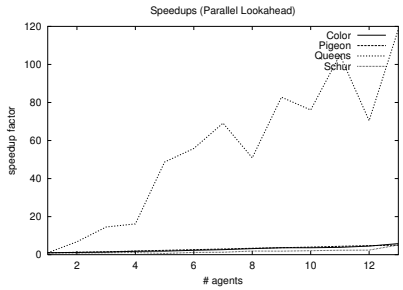

Fig. 14. Speedups for Parallel Lookahead

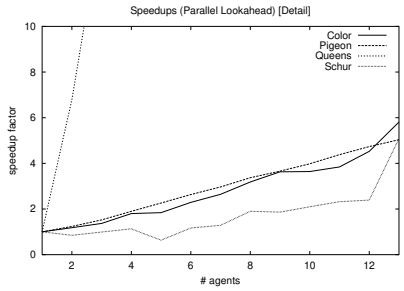

Fig. 15. Speedups for Parallel Lookahead

small amount of backtracking, the engine was run so that Horizontal Parallelism was given a higher priority than Vertical Parallelism by acting on the number of agents among which the set of unexplored literals is split. The experiments show, in general, a good speedup for all programs. The speedup, for 13 processors, is 5 for `schur` and `pigeon`, almost 6 for `color`, and 120 for `queens`. The speedup measured for `queens` is indeed surprising. It is interesting to note that `queens` requires (with *smodels*) the highest amount of backtracking. We conjecture that the speedup observed is the result of the combined application of both types of parallelism. However this issue deserves further investigation before any precise statement can be made. The results are definitely encouraging if we consider that: to the best of our knowledge, our system is one of the first exploiting Horizontal Parallelism; the way parallelism is handled is still very primitive if compared with the other existing parallel systems; the level of refinement of the algorithms for the computation of answer sets is still far beyond *smodels* (we expect the optimizations exploited in *smodels* to significantly improve speedup).

6.3 Current Research Directions

The current research has highlighted the inherent difficulties in the efficient exploitation of horizontal parallelism from ASP programs. The experiments conducted indicates that exploitation of horizontal parallelism is heavily hampered by some key aspects of ASP execution:

1. *granularity:* the steps performed during the execution of the `expand` operation are very fine grained;
2. *dependencies:* the activities required to expand a partial model require intense interactions—each worker needs to have an up-to-date view of the partial answer set in order to effectively progress the expansion;
3. *irregularity:* traditional partitioning techniques (e.g., partitioning based on predicates) lead to unbalanced computations and/or increased communication overheads.

In the search for better solutions to this problem, we have initiated an investigation aimed at developing horizontal parallel models for ASP where partitioning is driven by the syntactic and semantic properties of the program. This effort is facilitated by the rich collection of theoretical results that have been developed over the years in the context of stable models and answer set semantics.

We are currently exploring one main property of answer set programs to drive horizontal parallel execution: *splitting*.

Splitting is a property of logic programs under the answer set semantics originally studied by Lifschitz and Turner [28]. Given a logic program P, a *splitting set* of P is a set of atoms U with the following property: for each rule r in the program, if the head of r belongs to U, the all the atoms in the body of r belong to U. A splitting set U of P suggests a partitioning of the program in two parts: the set of all the rules whose head is in U (*bottom*—$b_U(P)$) and the set of all rules whose head is not in U (*top*—$t_U(P)$). The *Splitting Theorem* [28] guarantees that each answer set of P can be computed by first computing each answer set of $b_U(P)$ and then using such answer sets to determine the answer sets of $t_U(P)$. Splitting can be generalized to obtain a splitting of a program in n layers with the same property.

Our current effort is aimed at viewing the computation of answer sets as a *pipelined computation*, where the different stages of the pipeline corresponds to the different components of a program splitting. The pipeline allows data movement in both directions, since each layer of the pipeline can support the computation of both the preceding as well as the consecutive layers.

We are currently developing a prototypical implementation of an engine based on this view of horizontal parallelism. We expect that this approach can be significant for large size programs with a regular splitting structure—this is, for example, the case of answer set programs obtained from planning applications [5].

7 Other Issues: Parallel Grounding

7.1 Parallelizing Lparse

The first phase of the execution is characterized by the grounding of the input program. Although most interesting programs invest the majority of their execution time in the actual computation of models, the execution of the local grounding can still require a non-negligible amount of time. For this reason we have decided to investigate simple ways to exploit parallelism also from the preprocessing phase.

The structure of the local grounding process, as illustrated in [46], is based on taking advantage of the strong range restriction to individually ground each rule in the program. The process can be parallelized by simply distributing the task of grounding the different rules to different agents, as in Fig. 16. The **forall** indicated in the algorithm represents a parallel com-

putation: the different iterations are independent of each other. The actual solution adopted in our system is based on the use of a *distribution function* which statically computes a partition of the program Π (after removing all rules defining the domain predicates) and assigns the elements of the partition to the available computing agents. The choice of performing a static assignment is dictated by *(i)* the large amount of work typically generated, and *(ii)* the desire to avoid costly dynamic scheduling in a distributed memory context. The various computing agents provide as result the ground instantiations of all the rules in their assigned component of the partition of Π. The partitioning of Π is performed in a way to attempt to balance the load between processors. The heuristic used in this context assigns a weight to each rule (an estimation of the number of instances based on the size of the relations of the domain predicates in the body of the rule) and attempts to distribute balanced weight to each agent. Although simplistic in its design, the heuristics have proved effective in the experiments performed.

function ParallelGround(Π)
  $\Pi_G = \{a \mid$ *a is instance of domain predicate*$\}$
  $\Pi = \Pi \setminus \Pi_G$
  forall $R^i \in \Pi$
    $R^i_G =$ `GroundRule`(R^i)
  endall
  $\Pi_G = \bigcup R^i_G$
end

Fig. 16: Parallel Preprocessing

The preprocessor has been implemented as part of our ASP system, and it is designed to be compatible in input/output formats with the *lparse* preprocessor used in *smodels*. The preprocessor makes use of an internal representation of the program based on structure sharing—the input rule acts as skeleton and the different instantiations are described as environments for such skeleton. The remaining data structures are essentially identical to those described for the *lparse* system [46]. The implementation of the preprocessor, developed on a Beowulf system, has been organized as a master-slave structure, where the master agent is in charge of computing the program partition while the slaves are in charge of grounding the rules in each partition.

7.2 Experimental Results

We have analyzed the performance of the parallel preprocessor by comparing its execution speed with varying number of processors. The parallel preprocessor is in its first prototype and it is very unoptimized (compared to *lparse* we have observed differences in speed ranging from 4% to 48%). Nevertheless, the current implementation was mostly meant to represent a proof of concept concerning the feasibility of extracting parallelism from the preprocessing phase.

The first interesting result that we have observed is that the rather embarrassingly parallel structure of the computation allowed us to make the parallel overhead (i.e., the added computation cost due to the exploitation of parallelism) almost negligible. This can be seen in Fig. 17, which compares the execution times for a direct sequential implementation of the grounding algorithm with

the execution times using a single agent in the parallel preprocessor. In no cases we have observed overhead higher than 4.1%. Very good speedups have been observed in each benchmark containing a sufficient number of rules to keep the agents busy. Fig. 18 shows the preprocessing time for two benchmarks using different numbers of processors. Note that for certain benchmarks the speedup is slightly lower than linear due to slightly unbalanced distribution of work between the agents—in the current scheme we are simply relying on a static partitioning without any additional load balancing activities.

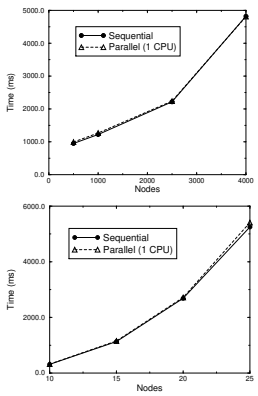

Fig. 17. Preprocessing Overhead (Pigeon, Coloring)

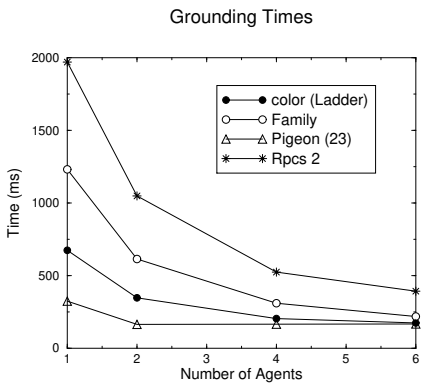

Fig. 18. Parallel Execution of the Preprocessor

8 Conclusions

In this paper we have presented an overview of the current effort in developing technology for the parallel execution of Answer Set programs. ASP has quickly become a leading paradigm for the high-level development of applications in areas such as planning and scheduling.

The investigation has lead to the identification of two major forms of parallelism—horizontal parallelism and vertical parallelism—that can be automatically exploited from the commonly used execution model for ASP. We have illustrated the major issues behind the exploitation of these forms of parallelism and described some possible solutions. These solutions have been integrated in actual prototypes and the paper reported the performance results obtained.

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