

# Computational Logic

## Concurrent (Constraint) Logic Programming

## Concurrent Logic Programs

---

- Predicate: Set of clauses
- Clause:  $Head \text{ :- } Guard \mid Body.$ 
  - ◇ *Head* is an atom
  - ◇ *Guard* and *Body* are conjunctions of atoms
- Resolvent: Set of goals (instances of atoms)
- Operational semantics: rewrite a goal in the resolvent with one of the clauses in the matching predicate definition
- Concurrency:
  - ◇ “No” goal selection rule (i.e., concurrent selection rule)
  - ◇ “No” clause search rule (i.e., concurrent search rule)

## Synchronization Rules

---

- Clause matching: *Head + Guard*.
  - ◇ *Head* matches the goal
  - ◇ *Guard* is successful
- A head matches a goal if the goal is an instance of the head
- A guard is executed in one-way unification mode
- Suspension: if a head does not match the goal, but it could do so in the future, then it *suspends*

## An Example

---

$p(X) :- X = a \mid r.$

$p(X) :- X = b \mid s.$

$q(X) :- \text{true} \mid X = b.$

$?- p(X), q(X).$

- There is no ordering in the execution of  $\langle p(X), q(X) \rangle$
- There is no ordering in the execution of clauses of  $p(X)$
- Clauses of  $p(X)$  suspend
- The clause of  $q(X)$  continues (“commits”)
- Then,  $q(X)$  instantiates  $\{X/b\}$  in the body
- The second clause of  $p(X)$  continues (“commits”), while first clause fails.

## Logic vs. Concurrent Logic Programming

---

- The logical variable as a communication channel

Logic	Concurrent Logic
shared logical variable	communication channel
instantiation	communication
head unification	synchronization

- Unification Revisited:
  - ◇ One-way (Read-only) unification — Ask
    - \* in *Head* and in *Guard*
  - ◇ Two-way (Output) unification — Tell
    - \* only in *Body*
  - ◇ Suspension:
    - \* Due to read-only unification in clause selection

## Logic vs. Concurrent Logic Programming

---

- Committed-choice: clause selection is irrevocable
- No backtracking allowed

Logic	Concurrent Logic
cut	commit
“don’t know” non-determinism	(“don’t care” non-determinism) indeterminism
search	selection

- Guards:
  - ◇ Flat guards: only selected predicates in guards
    - \* (Special) builtins
    - \* Possibly also facts
  - ◇ Deep guards: calls to any predicate allowed in guards
    - \* User-defined predicates, too

## Logic vs. Concurrent Logic Programming

- Goals as processes:

Logic	Concurrent Logic
atomic goal	process
goal (set of atoms)	process network
clause	process instruction

- Process Behaviour:

- ◇ Change state of process network:

- \* Become a new process:

$$A :- G \mid B.$$

- \* Become  $k$  concurrent processes:

$$A :- G \mid B_1 \dots B_k.$$

- ◇ Halt:

$$A :- G \mid true.$$

- ◇ Change state of data:

$$A :- G \mid \dots A.$$

- Some syntactic sugar:

- ◇  $A :- G \mid true. \Leftrightarrow A :- G \mid .$

- ◇  $A :- true \mid G. \Leftrightarrow A :- \mid G. \Leftrightarrow A :- G.$

- ◇  $A :- true \mid true. \Leftrightarrow A.$

## Process Behaviour Examples

---

- Become a new process:  $A :- G \mid B.$

$p(X) :- X=f(a,Y) \mid q(Y).$

- Become  $k$  concurrent processes:  $A :- G \mid B_1 \dots B_k.$

$p(X) :- X=f(A,B,C) \mid q(A), r(B), s(C).$

- Halt:  $A :- G \mid .$

$p(X) :- X=f(a) \mid .$

- Change state of data:  $A :- G \mid \dots A.$

$p(X) :- X=f(a,Y) \mid Y=f(b,Z), p(Z).$

$p(I,S) :- I=[H|NI], int(H) \mid NS \text{ is } S+H, p(NI,NS).$



## Incomplete Messages

---

- Back-communication:

`?- q(X), p(X).`

`p(X):- X=f(a,Y), check(Y).`

`check(ok).`

`q(f(X,Y)):- X=a | Y=ok.`

## Incomplete Messages (Contd.)

---

- Dialogue:

```
?- q(X), p(more(X)).
```

```
p(more(X)):- X=f(a,Y), p(Y).
```

```
p(more(X)):- X=f(b,Y), p(Y).
```

```
p(ok).
```

```
q(f(X,Y)):- X=b | Y=more(Z), q(Z).
```

```
q(f(X,Y)):- X=a | Y=ok.
```

- Network formation and reconfiguration:

```
?- q(A), p(A).
```

```
p(A):- A=channels(X,Y,Z), p1(X), p2(Y), p3(Z).
```

```
q(channels(X,Y,Z)):- q1(X), q2(Y), q3(Z).
```

## The Logical Variable

---

- A shared variable acts like:
  - ◇ A communication channel to send a message
  - ◇ A shared location being accessed concurrently
- Equivalences/conceptual view:
  - ◇ One shared variable = One message
  - ◇ Instantiation = Sending a message
  - ◇ Partially instantiated term = incomplete message = open channel
  - ◇ Ground term = complete message = closed channel
  - ◇ Recursive term = stream of messages
- Incomplete structures: an incomplete message can be thought of as:
  - ◇ A message being incrementally sent
  - ◇ An open communication channel
  - ◇ A message with sender's identity
  - ◇ A structure being co-operatively constructed

## Streams of Messages

---

- A stream producer

```
naturals(N,Is):- Is=[N|Is1], N1 is N+1, naturals(N1,Is1).
```

- A stream consumer

```
sum([N|Is],Tmp,Sum):- N>=0 | TN is Tmp+N, sum(Is,TN,Sum).
```

- Producer/Consumer (asynchronous)

```
?- naturals(0,I), sum(I,0,Total).
```

- Producer/Consumer on demand (synchronous)

```
?- naturals(0,I), sum(I,0,Total), I=[_|_].
```

```
naturals(N,[I|Is]):- I=N, N1 is N+1, naturals(N1,Is).
```

```
sum([N|Is],Tmp,Sum):- N>=0 | Is=[_|_], TN is Tmp+N, sum(Is,TN,Sum).
```

- Key issue: who produces the buffer?

## Merging and Dispatching Streams

---

- A stream merger:

```
merge([X|Xs],Ys,Out):- Out=[X|Zs], merge(Xs,Ys,Zs).
```

```
merge(Xs,[Y|Ys],Out):- Out=[Y|Zs], merge(Xs,Ys,Zs).
```

```
merge([],Ys,Out):- Out=Ys.
```

```
merge(Xs,[],Out):- Out=Xs.
```

- A (copying) stream dispatcher?

```
dispatch([X|Xs],Out1,Out2):- Out1=[X|Ys], Out2=[X|Zs], dispatch(Xs,Ys,Zs).
```

```
dispatch([],Out1,Out2):- Out1=[], Out2=[].
```

- A (caotic) stream dispatcher:

```
dispatch([X|Xs],Out1,Out2):- Out1=[X|Ys], dispatch(Xs,Ys,Out2).
```

```
dispatch([X|Xs],Out1,Out2):- Out2=[X|Ys], dispatch(Xs,Out1,Ys).
```

```
dispatch([],Out1,Out2):- Out1=[], Out2=[].
```

- A stream dispatcher with senders' identities

```
dispatch([mess(1,X)|Xs],Out1,Out2):- Out1=[X|Ys], dispatch(Xs,Ys,Out2).
```

```
dispatch([mess(2,X)|Xs],Out1,Out2):- Out2=[X|Ys], dispatch(Xs,Out1,Ys).
```

```
dispatch([],Out1,Out2):- Out1=[], Out2=[].
```

## Fairness

---

“An event that may occur will eventually occur”

- Or-Indeterminism: clause selection  $\Rightarrow$  Or-Fairness (clauses eventually selected)
- And-Indeterm.: goal reduction  $\Rightarrow$  And-Fairness (allows non-terminating procs.)
- A stream merger:

```
merge([X|Xs],Ys,Out):- Out=[X|Zs], merge(Xs,Ys,Zs).
merge(Xs,[Y|Ys],Out):- Out=[Y|Zs], merge(Xs,Ys,Zs).
merge([],Ys,Out):- Out=Ys.
merge(Xs,[],Out):- Out=Xs.
```

Key: or-fairness required, otherwise it is just append!

- An eager producer:

```
naturals(N,Is):- | Is=[N|Is1], N1 is N+1, naturals(N1,Is1).

?- naturals(0,I), sum(I,0,Total).
```

Key: and-fairness required, otherwise nothing is ever consumed!

## Termination Issues

---

- Non-terminating (but running) processes:

```
?- naturals(I), sum(I,Total), I=[_|_].
```

```
naturals(I):- naturals(0,I).
```

```
naturals(N,[I|Is]):- | I=N, N1 is N+1, naturals(N1,Is).
```

```
sum(I,Total):- sum(I,0,Total).
```

```
sum([N|Is],Tmp,Sum):- N>=0 | Is=[_|_], TN is Tmp+N, sum(Is,TN,Sum).
```

## Termination Issues (Contd.)

---

- Deadlock:

?- q(X), p(X).

p(more(X)) :- X=f(a,Y), p(Y).

p(more(X)) :- X=f(b,Y), p(Y).

p(ok).

q(f(X,Y)) :- X=b | Y=more(Z), q(Z).

q(f(X,Y)) :- X=a | Y=ok.



## Bounded-Size Communication Media

---

- Producer/Consumer with fixed sized communication (e.g., size=4) and termination:

```
?- naturals(0,I), sum(I,0,Total), I=[_1,_2,_3,_4].
```

```
naturals(N,[I|Is]):- | I=N, N1 is N+1, naturals(N1,Is).  
naturals(N,[]).
```

```
sum([N|Is],Tmp,Sum):- N>=0 | TN is Tmp+N,sum(Is,TN,Sum).  
sum([],Tmp,Sum):- | Sum=Tmp.
```

Key: the communication media is produced from outside and fixed size!

- Dynamically-sized media:

```
?- naturals(0,I), sum(I,0,Total), medium(4,I).
```

```
medium(0,Stream) :- Stream = [].  
medium(N,Stream):- N>0 |Stream=[_|Stream1], medium(N-1,Stream1).
```

## Bounded-Buffer Communication

---

- Bounded buffer:

```
buffer(0,Stream,Tail):- Stream=Tail.
```

```
buffer(N,Stream,Tail):- N>0 | Stream=[_|Stream1], buffer(N-1,Stream1,Tail).
```

Creates buffer as open list of N elements, passes handle to list end

- Simple producer with termination at Max elements:

```
naturals(N,[I|Is],Max):- N<=Max | I=N, N1 is N+1, naturals(N1,Is,Max).
```

```
naturals(N,I,Max):- N>Max | I=[].
```

Suspended until buffer available. Closes buffer at Max elements

- Consumer:

```
sum([N|Is],Tail,Acc,Sum):- N>=0 |
```

```
    Tail=[_|Tail1], NAcc is Acc+N, sum(Is,Tail1,NAcc,Sum).
```

```
sum([],Tail,Acc,Sum) :- Acc = Sum.
```

Suspended until buffer and element available. Adds one more element to the buffer for each element consumed.

- Usage (e.g., for buffer length = 18, termination at 1000 elements):

```
?- naturals(0,Buffer,1000), sum(Buffer,Tail,0>Total), buffer(18,Buffer,Tail)
```

## Bounded-Buffer Communication (Contd.)

---

- Overall effect is still asynchronous!
- Producer can get ahead of consumer by a fixed number of elements. After that, suspended on stream until Consumer requests more.

## Streams of Messages: Protocols

---

- One-to-one communication:  
One producer + One consumer
- Duplex communication:  
Two producer/consumers
- Broadcast communication:  
One producer + Many consumers
- Many-to-one communication:  
Many producers + One consumer
- Blackboard communication:  
Many producers + Many consumers:  
Many producers/consumers

## Broadcast Communication

---

- Matrix multiplication:

```
?- vector(V), matrix(M), vm(V,M,Result).
```

```
vm(_, [], Zv) :- Zv = [].
```

```
vm(Xv, [Yv|Ym], Zv) :- Zv = [Z|Zv1],  
    vv(Xv, Yv, Z),  
    vm(Xv, Ym, Zv1).
```

```
vv(Xv, Yv, P) :- vv1(Xv, Yv, 0, P).
```

```
vv1([], [], S, P) :- P = S.
```

```
vv1([X|Xv], [Y|Yv], S, P) :- S1 is S+X*Y |  
    vv1(Xv, Yv, S1, P).
```

- Broadcasting of  $V$  to all  $vv/3$  processes
- Dynamically configured network of  $vv/3$  processes

## Many-to-one Communication

---

- A data abstraction: queues

```
queue([dequeue(X) | S], Head, Tail) :-  
    Head = [X | NewHead],  
    queue(S, NewHead, Tail).  
queue([enqueue(X) | S], Head, Tail) :-  
    Tail = [X | NewTail],  
    queue(S, Head, NewTail).  
queue([], _, _).
```

## Many-to-one Communication (Contd.)

---

- A simulator of a multiprocessor machine

```
?- processors(10,Job), Job=...
```

```
processors(N,X):-  
    queue(S,[X|Xs],Xs),  
    processors(1,N,S).
```

```
processors(N,N,S):-  
    processor(N,idle,S).  
processors(N1,N4,S):-  
    N2 is (N1+N4)/2 | N3 is N2+1,  
    processors(N1,N2,S1),  
    processors(N3,N4,S2),  
    merge(S1,S2,S).
```

- N processor/3 proc. communicating with one queue/3 proc.
- Statically configured network of proc.: spawning / computing phases (“systolic”)

## Many-to-many Communication

---

- A network of producers and consumers

```
?- consumers(Buffer), producers(Buffer).
```

```
producers(Stream):- p1(X), p2(Y), p3(Z),  
                    merge(X,Y,Stream1), merge(Z,Stream1,Stream).
```

```
consumers(Stream):- c1(Stream), c2(Stream), c3(Stream).
```

```
p1(S):- S=[message(1,Mess)|Xs], produce(Mess), p1(Xs).  
p1(S):- S=[].
```

```
c1([X|Xs]):- X=message(1,Mess) | consume(Mess), c1(Xs).  
c1([X|Xs]):- X=message(Id,Mess), Id=\=1 | c1(Xs).  
c1([]).
```

- Blackboard Communication:
  - ◇ Needed driver for the blackboard



## Operational Semantics

---

- Rewriting system

$$\text{match}(A, A') = \begin{cases} \theta & \text{if } A = A'\theta \text{ } mgu(A, A') = \theta \\ \text{fail} & \text{if } mgu(A, A') = \text{fail} \\ \text{suspend} & \text{otherwise} \end{cases}$$

$$\text{try}(A, (A' \leftarrow G \mid B)) = \begin{cases} \theta & \text{if } \text{match}(A, A') = \theta \wedge \\ & \text{check}(G\theta) = \text{true} \\ \text{fail} & \text{if } \text{match}(A, A') = \theta \wedge \\ & \text{check}(G\theta) = \text{fail} \vee \\ & \text{match}(A, A') = \text{fail} \\ \text{suspend} & \text{otherwise} \end{cases}$$

## Operational Semantics (Contd.)

---

- **Reduction:**  $A_1 \dots A_i \dots A_n; \theta \rightarrow (A_1 \dots B_1 \dots B_k \dots A_n) \theta'; \theta \circ \theta'$   
if  $\exists C = A \leftarrow G \mid B_1 \dots B_n$  s.t.  $try(A_i, C) = \theta'$
- **Failure:**  $A_1 \dots A_i \dots A_n; \theta \rightarrow fail; \theta$   
if  $\forall C \ try(A_i, C) = fail$
- **Guard checking:**
  - ◇ Flat guards: use *match* in all unifications
  - ◇ Deep guards: copy environment

## (Some) Concurrent Logic Languages

---

- Parlog [Clark, Gregory 83]
  - ◇ mode declarations for input/output arguments
  - ◇ safe clauses: output instantiation in guards is an error
  - ◇ one-way unification in guards
- Concurrent Prolog [Shapiro 84]
  - ◇ read-only annotation of variables in calls
  - ◇ local environments for guards
  - ◇ atomic extended head unification
- GHC (Guarded Horn Clauses) [Ueda 85]
  - ◇ different interpretation of unification in guard and body
  - ◇ suspension on output instantiation in guards
  - ◇ general unification with guard restriction

## (Some) Concurrent Logic Languages (Contd.)

---

- Implementation Issues:
  - ◇ Parlog
    - \* compile-time safety check
  - ◇ Concurrent Prolog
    - \* support for local environments
    - \* detection of inconsistency with global environment
  - ◇ GHC
    - \* identification of variables on which to suspend
- Problems: no backtracking.
- More Recent Systems:
  - ◇ Andorra-I: only deterministic computations proceed.
  - ◇ AKL: goals execute in a local environment.
  - ◇ BinProlog: communication through blackboard.
  - ◇ CIAO: communication through shared database.